PAPER

Enhanced Resonance by Coupling and Summing in Sinusoidally Driven Chaotic Neural Networks

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SUMMARY Enhancement of resonance is shown by coupling and summing in sinusoidally driven chaotic neural networks. This resonance phenomenon has a peak at a drive frequency similar to noise-induced stochastic resonance (SR), however, the mechanism is different from noise-induced SR. We numerically study the properties of resonance in chaotic neural networks in the turbulent phase with summing and homogeneous coupling, with particular consideration of enhancement of the signal-to-noise ratio (SNR) by coupling and summing. Summing networks can enhance the SNR of a mean field based on the law of large numbers. Global coupling can enhance the SNR of a mean field and a neuron in the network. However, enhancement is not guaranteed and depends on the parameters. A combination of coupling and summing enhances the SNR, but summing to provide a mean field is more effective than coupling on a neuron level to promote the SNR. The global coupling network has a negative correlation between the SNR of the mean field and the Kolmogorov-Sinai (KS) entropy, and between the SNR of a neuron in the network and the KS entropy. This negative correlation is similar to the results of the driven single neuron model. The SNR is enhanced around a few frequencies and the dependence on frequency is clearer and smoother than the results of the driven single neuron model. The SNR is saturated as an increase in the drive amplitude, and further increases change the state into a nonchaotic one. Such dependence on the drive frequency and amplitude exhibits similarities to the results of the driven single neuron model. The nearest neighbor coupling network with a periodic or free boundary can also enhance the SNR of a neuron depending on the parameters. The network also has a negative correlation between the SNR of a neuron and the KS entropy whenever the boundary is periodic or free. The network with a free boundary does not have a significant effect on the SNR from both edges of the free boundaries.

key words: driven chaotic system, chaotic neural network, biological information processing, stochastic resonance, enhancement, coupling

1. Introduction

In a biological system, information processing can be adapted to the environment. This adaptability is investigated considering nonlinearity and cooperation among nonlinear elements in science and engineering. Recently, neurocomputing studies have deepened our understanding of information processing in biological nerve system. Chaotic dynamics in actual neural networks and their models have received considerable attention in this field. In actual biological systems, the existence of chaos is reported by analyses of various time series observed macroscopically and microscopically [1]–[4]. From these results, we can guess that chaos plays an important role in processing information in a biological system.

In nonlinear science, stochastic resonance (SR) has recently received considerable attention [5]–[7]. SR is considered as an nonlinear effect that is used in detecting a weak signal from a real noisy environment by the biological nerve systems. In SR, background noise yields the benefit of weak signal detection in some nonlinear systems, such as a bistable system. The basic SR mechanism is that the system is synchronized and resonated by an external periodic signal when a bistable system is driven by a subthreshold sinusoid plus some amount of noise. The subthreshold sinusoid alone can not overcome the potential barrier of a bistable system, but the addition of noise helps this movement between wells of bistable potential. This timing is occasionally synchronized to an external periodic signal. The power spectrum of the system's time series has peaks at the drive-signal frequency and its harmonics. Suppose that a detector can catch the signal when the system's state overcomes the potential barrier. The power spectrum of the external signal, which contains the sinusoid and noise has no peak at the sinusoid frequency. In a SR situation, the subthreshold sinusoid can be found as a peak with background noise in the power spectrum of a detector's response. This indicates that SR enhances the detection of a weak signal from background noise. SR is also observed in threshold systems, but the result is a noise-induced crossing of the threshold rather than resonance, because the system is independent of the drive signal frequency [8], [9]. Excitable media also show SR [10]. For these reasons, it has been shown that neuron models such as the Fitzhugh-Nagumo model [11], [12], exhibit SR behaviors [13].

This SR effect is found in the actual sensory neurons, such as a mechanoreceptive sensory neuron from crayfish [14], rat skin [15], and an interneuron from cricket abdominal ganglia [16]. This effect obviously provides a useful sensory function for the segmentation of noisy backgrounds. The neural network of a biological system has been investigated with respect to SR. The extracel-

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lular potential of hippocampal slices in a rat temporal lobe shows SR in electric fields of a sinusoid plus noise [17]. The mechanism has also been introduced to explain fluctuation in the human visual perception in resolving ambiguous figures [18], for example, known as the Necker cube [19] in visual psychology. In biological systems, studies on SR are considered to give an understanding of the sensory mechanisms in real noisy environments and high-level cognitive processes.

SR is known as an effect driven by two cooperative signals, i.e., a sinusoid and noise [5]–[7]. This situation considered as a perturbed system is more complex than that for one with a driving-signal. In this paper, we consider a chaotic system as a fluctuating system instead of a bistable system driven by noise. There has been a lot of studies on nonlinear systems perturbed by adding a signal or by multiplying a parameter by a signal. However, few papers [20]–[23] have addressed perturbed chaotic systems in the context of SR. The SR effect in a noiseless deterministic chaotic system is observed when the bifurcation parameter is driven periodically in time below and above the threshold for the onset of chaos. The mechanism of this resonance is the same as in an excitable system driven near a bifurcation point. SR is also found at the bifurcation point of intermittency where a chaos attractor separates into two attractors. SR is found in bistable systems of not only fixed points but also any kind of attractor, even chaos. Chaos spontaneously generates its own effective noise.

We also do not consider any external stochastic forces in this paper, but similar to previous research, we regard a chaotic fluctuation as stochastic noise in the studied system. Note that the ratio of an input signal to an input noise for these systems is infinite because the external input is a noise-free sinusoid. In this paper, according to previous studies, we attach great importance to understanding the resonance properties, and not to detecting a weak signal as a noise-induced effect. Stochastic noise is generated intrinsically from the system as chaos. If we can consider intrinsic spontaneous noise as deterministic chaos, the nerve system in a biological system may benefit from the resonance generated in driven chaotic systems.

Previously, members of our group reported a resonance phenomenon in a chaotic neuron model driven by a sinusoid without noise [24]. The chaotic neuron model and its network were proposed by Aihara et al. and are based on their studies of squid giant axons and the Hodgkin-Huxley equation [25]. The networks of the neuron model have been intensively studied in information processing, with concentration on escaping from local minima in optimization and retrieving associative memory [26]–[28]. Recently, some chaos control techniques have also been applied to networks to stabilize the chaotic retrieval process [29], [30]. The neuron model is a generalization of the McCulloch-Pitts model [31] as a threshold system, or the Fitzhugh-Nagumo model [11], [12] as an excitable system. This model has also become a chaotic system. We showed that, similar to SR in the bistable case, the chaotic neuron model driven by a weak sinusoid also has obvious peaks in the power spectrum [24]. The signal-to-noise ratio (SNR) depends on not only the Lyapunov exponent but also the phase structure of chaotic attractors. We cannot merely regard the Lyapunov exponent as an intensity of noise in noise-induced SR. However, there is a negative correlation between the SNR and the Lyapunov exponent. The resonance mechanism is different from noise-induced SR because of the lack of chaotic multi-attractors. The perturbation can be regarded as a slow change of a bifurcation parameter in a chaotic regime. When the phase structure of an attractor is sensitive to this parameter, the slow perturbation - slow compared to the time scale of the attractor - derives a resonance phenomenon.

In this paper, we consider the resonance in chaotic neural networks to study a coupling and summing effect. We investigate properties of the resonance in the networks and the effects of coupling and summing on resonance . The enhancement of SNR by coupling or summing elements has been recently studied in SR in extended systems [10], [32]–[34]. The researchers reported that coupling or summing produces a significant enhancement of the response due to a cooperative effect in spite of some differences among the driven systems. There is also a difference between the observed measures, for example, the output of a single element or a mean field of the system. Here, we note that a mean field of neuron internal states and one of neuron internal states are used to calculate the SNR in chaotic neural networks. We also note that the chaotic neural network is driven by a weak sinusoid alone, which does not include noise as noise induced SR.

2. Chaotic Neural Networks

The original chaotic neuron model is represented as the dynamics of the difference equations of three variables [25]. A certain parameter combination can reduce variables into one. The simplest chaotic neuron model has the following dynamics.

$$\begin{aligned} y(t+1) &= ky(t) - \alpha f(y(t)) + a, \\ x(t+1) &= f(y(t+1)), \\ f(z) &= \frac{1}{1+\exp(-\frac{z}{\epsilon})}, \end{aligned} \tag{1}$$

where
$y(t)$: internal state of a neuron at time t,
$x(t)$: output of a neuron at time t,
k : damping constant,
α : refractory parameter ($\alpha \geqq 0$),

a : parameter based on the input and the threshold,
ϵ : steepness parameter of the sigmoid function.
We call parameter a the input as a bifurcation parameter. The bifurcation diagram and the Lyapunov exponent appeared in Ref. [25], [28], [35]. We fix the parameter set at $k = 0.7$, $\alpha = 1.0$, and $\epsilon = 0.02$ throughout this paper. We study simple coupling networks with a homogeneous input. The networks driven by a weak sinusoid can be represented by the following equation.

$$\begin{aligned} y_i(t+1) &= ky_i(t) - \alpha f(y_i(t)) \\ &\quad + \sum_j w\{f(y_j(t)) - f(y_i(t))\} + a \\ &\quad + A\sin 2\pi f_m t, \\ x_i(t+1) &= f(y_i(t+1)). \end{aligned} \tag{2}$$

Here, w represents the weight in the network, A is the amplitude of the drive signal, and f_m is the frequency. In Eq. (2), $\sum_j$ means $\sum_{j=1}^{N}$ for a global coupling and $\sum_{j=i-1}^{j=i+1}$ for the nearest neighbor coupling with a periodic or free boundary condition. A periodic boundary means that $y_0(t) = y_N(t)$, $y_{N+1}(t) = y_1(t)$ and a free boundary means that $y_0(t) = y_1(t)$, $y_{N+1}(t) = y_N(t)$. We note that in Eq. (2) the i-th neuron has interactions of the difference between own neuron and the others. This formulation corresponds that the original chaotic neural network [25] has the refractory parameter as $\alpha + N_{sum}w$. Here, N_{sum} is the number of neurons which the i-th neuron interacts. Because we can transform the first equation of Eq. (2) into the following equation.

$$\begin{aligned} y_i(t+1) &= ky_i(t) - (\alpha + N_{sum}w)f(y_i(t)) \\ &\quad + \sum_j wf(y_j(t)) + a \\ &\quad + A\sin 2\pi f_m t. \end{aligned} \tag{3}$$

This formulation is introduced to scale the refractory parameter corresponding to varying the weight. We normally consider $N = 100$ networks and use $A = 0.005$ and $f_m = 0.125$ for the drive parameters for the chaotic regime, unless otherwise mentioned.

3. Fundamental Mechanism of Stochastic Resonance and Enhanced Response by Coupling or Summing Elements

SR is a noise-induced effect in nonlinear systems [5]–[7]. Consider an overdamped ball $x(t)$ moving in double-well potential $U(x)$ such that

$$\frac{dx}{dt} = -\frac{\partial U}{\partial x} + F(t) + A\sin 2\pi f_0 t \tag{4}$$

with a weak time-periodic signal, $A\sin 2\pi f_0 t$, and noise $F(t)$. The ball is weakly forced sinusoidally in time, but can not overcome the potential barrier ΔU with a weak periodic signal alone. A certain amount of force is required for the ball to overcome the barrier. If we add noisy fluctuating force $F(t)$, the ball can occasionally jump over the barrier. Here, $F(t)$ is Gaussian white noise $[\langle F(t)\rangle = 0, \langle F(t)F(t')\rangle = 2D\delta(t-t')]$. The chance of switching the wells is synchronized with the amplitude peak of the periodic signal. Time series $x(t)$ of incidents, where the barrier is overcome, has peak $S(f_0)$ at sinusoid frequency f_0 in the power spectrum. This peak does not appear when the noise is excessively small or excessively large. This is because in the former case the incidents (where the barrier is overcome) do not occur, and in the latter they occur at random. There is an optimum amount of noise to detect the weak signal for any given SNR [for example, SNR$= 10\log_{10}(S(f_0)/N)$]. The mechanism of SR is not directly related to resonance phenomena in chaotic systems, but it may be useful for understanding the resonance mechanism in chaotic systems.

SR in coupled bistable elements has recently been studied for enhanced SNR by coupling or summing elements [10], [32]–[34]. These systems are comprised of coupled nonlinear elements perturbed by a weak periodic sinusoid in a noisy background. The situations studied are similar, for example, the elements ordinarily have nonlinearity and the sinusoid is identical to each element. However, there are some differences in that the system has coupling or summing among the elements. The coupling is linear or nonlinear, local or global, and the noise is independent of each element or the same for each element. Jung and Mayer-Kress presented the first spatiotemporal patterns that show SR in an excitable medium [10]. The system has a linear local coupling among threshold elements and noise is independent of each element. In the two-dimensional driven array of the element, spiral waves are formed depending on the external noise and coupling strength. When the array is driven by a solitary wave, the synchronization of the firing pattern to the driving shows a resonance peak as a function of the noise strength.

Inchiosa and Bulsara investigated a network with a globally and nonlinearly coupled Hopfield-type neuron model [32]. They reported that the system driven by an identical sinusoid and noise for each element significantly enhances the system's collective response over that of a single neuron. Their system does not have identical elements and one element used as "a reference" is bistable, and the remaining elements used as "a heat bath" are all either monostable or bistable. Using the adiabatic assumption and the slaving principle [36], the dynamics of N globally coupled neurons was reduced to single neuron dynamics in a potential. The SNR of this single neuron as the reference is enhanced by nonlinear coupling. Lindner et al. considered a 1-dimensional chain of the Duffing oscillators with the nearest neighbor coupling [33]. They reported that the linear coupling of nonlinear oscillators significantly en-

hances the SNR of the response of a single oscillator subject to an identical time-periodic signal and independent noise to each oscillator.

Collins et al. suggested a summing network which is subject to independent noise of each element and the network enhances its response by summing [34]. They used aperiodic signal for the drive signal and evaluated the correlation between an input signal and a mean field of outputs. This enhancement is independent of the input noise level. Even when the drive signal is periodic, this enhancement by summing is understood by the law of large numbers. They insist that the results suggest a possible functional role for neuronal noise in sensory systems from a neurophysiological viewpoint. All these studies reported an enhancement of a cooperative effect by coupling or summing, which was independent of each system condition, i.e., independent noise or identical noise for each element. In the next section, we examine such enhancement in chaotic neural networks as a chaotic coupled system.

4. Enhanced Resonance in the Chaotic Neural Networks

To study the properties of resonance in the chaotic neural network, we calculated the SNR from the power spectrum and the Kolmogorov-Sinai (KS) entropy from the Lyapunov spectrum to consider the dependence on the system parameters. We consider 4096 time steps for calculating the power spectrum by discarding 8192 transient time steps. We defined the SNR in time series as

$$\mathrm{SNR} = 10\log_{10}(\frac{S(f_m)}{N_p}). \tag{5}$$

Here, $S(f_m)$ is the peak power at drive signal frequency f_m and N_p is the noise power obtained by averaging 40 frequency points around the signal frequency. We use the time series of mean field $m(t)$ $(= \frac{1}{N}\sum_{i=1}^{N} y_i(t))$ of the internal states or internal state $y_i(t)$ of one neuron to calculate the SNR. The following reason is why we select internal state $y_i(t)$ and not output $x_i(t)$. In our studies of the driven chaotic neuron model [24], we found that for the sigmoid function, similar to threshold systems, the power spectrum of $x(t)$ has a higher peak than that of $y(t)$. We investigated the resonance of $y(t)$ rather than $x(t)$ to study the chaotic neuron model as a nonlinear map without the sigmoid function effect.

The KS entropy from the Lyapunov spectrum is also calculated corresponding to the SNR. The Lyapunov spectrum characterizes how a small disturbance in tangential space is amplified or contracted. There exists N exponents corresponding to N eigenvalues in each tangential vector. These form the Lyapunov spectrum which are ordered from the largest to smallest exponent. The sum of positive Lyapunov exponents represents the amplification ratio of tangential volume. This is called the KS entropy, which is originally defined as the increase ratio of a variety of symbol sequences. We consider 8192 time steps for calculating the Lyapunov spectrum by discarding 8192 transient time steps.

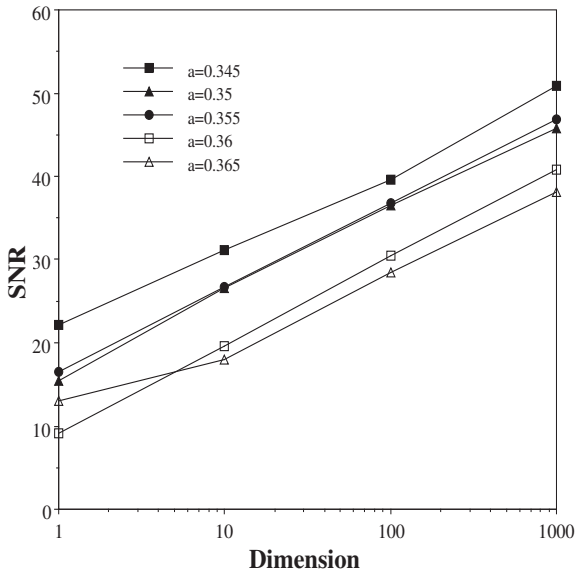
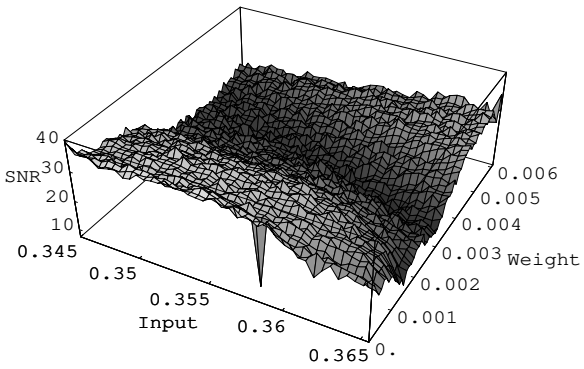


Fig. 1 SNR of mean field $m(t)$ as a function of network size N in a summing network.

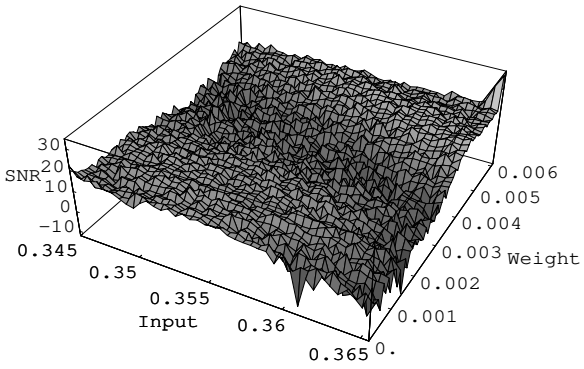
We set the networks' initial values $y_i(0)$ as random numbers $[0, 1]$. For a neuron parameter, we selected an input $a = [0.345, 0.365]$ region such that there is an increasing trend in the Lyapunov exponent of the unperturbed neuron model as in [24]. For a network parameter, we also selected a weight w region including almost all of the turbulent phase except for a few points considering the resonance phenomenon independent of each neuron in a network. The turbulent phase has the smooth Lyapunov spectrum, so that the phase is close to the independent chaotic behavior with no interaction between neurons. The considered network configurations have a global coupling and the nearest neighbor coupling with a periodic or free boundary. We note that the driven network by a weak sinusoid alone does not include stochastic noise as noise induced SR.

First, we consider the SNR of summing network as $w = 0$ and a global coupling network as $w > 0$. Figure 1 shows the dependence of the SNR of mean field $m(t)$ in a summing network against changing network size N. The SNR is enhanced when network size N increases [35]. This effect is understood by the law of large numbers in statistics as in Ref. [34]. We can theoretically find that all line slopes are 10 according to SNR definition in Eq. (5).

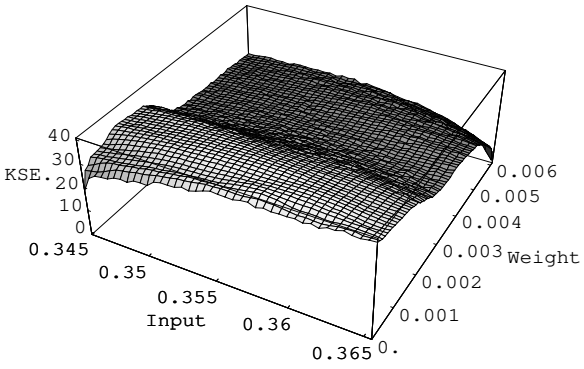
Figure 2 shows the dependence of the SNR of mean field $m(t)$ and one of neuron internal states $y_i(t)$, and the corresponding KS entropy on a network parameter,



(a) SNR of mean field $m(t)$.



(b) SNR of one of neuron internal states $y_i(t)$.



(c) Corresponding KS entropy.

Fig. 2 SNR of mean field $m(t)$ and one of neuron internal states $y_i(t)$ and corresponding KS entropy as a function of weight w and input a in a global coupling network.

weight w, and a neuron parameter, input a, in a global coupling network. The SNR and KS entropy are dependent on weight w and input a, but their relations are not simple. To clarify relations among the SNR values, we show the SNR of a mean field and the SNR of a neuron as a function of input a at fixed weight values in Fig. 3. We examine the following points from Fig. 2 and 3.

1. If we compare the SNR values of a mean field in $w > 0$ (global coupling network) and those at $w = 0$ (summing network), the parameter regions in larger values of the weight and input are such that the SNR values in $w > 0$ are superior to those at $w = 0$. There is enhancement by coupling in the mean field, but it is not always enhanced. The enhanced area on the mean field is small in these parameter regions of the turbulence phase. There are enhanced areas around the turbulence phase; however, the SNR value is strongly dependent on each neuron.

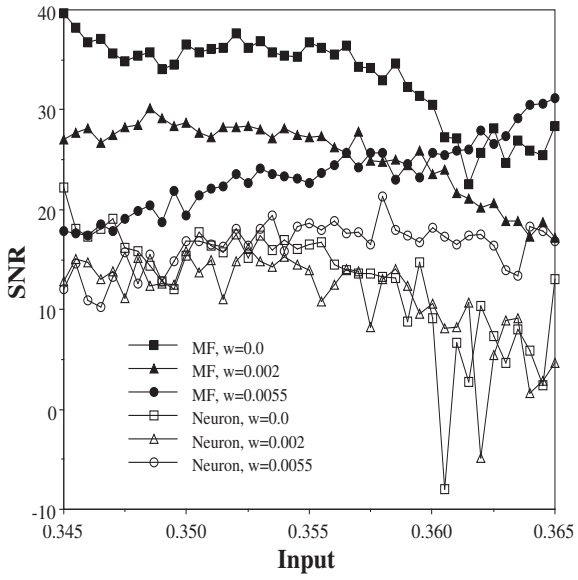


Fig. 3 SNR of mean field $m(t)$ (MF) and a neuron $y_i(t)$ (Neuron) as a function of input a at fixed weight values in a global coupling network.

2. If we compare the SNR values of a mean field and those of a neuron in $w > 0$ (global coupling network), the SNR values of a mean field are superior to those of a neuron. There is enhancement by summing in a coupling network. Almost all of the enhanced areas are in our calculated parameter regions.

3. If we compare the SNR values of a mean field in $w > 0$ (global coupling network) and those of a neuron at $w = 0$ (single neuron model), the parameter regions are such that the SNR values of a mean field are superior to those of a neuron. There is enhancement by a combination of coupling and summing. Almost of the enhanced areas are in our calculated parameter regions.

4. If we compare the SNR values of a mean field at $w = 0$ (summing network) and a neuron in $w \geq 0$ (global coupling network or single neuron model), the SNR values of a mean field are superior to those of a neuron. Thus, summing to provide a mean field is more effective than coupling on a neuron

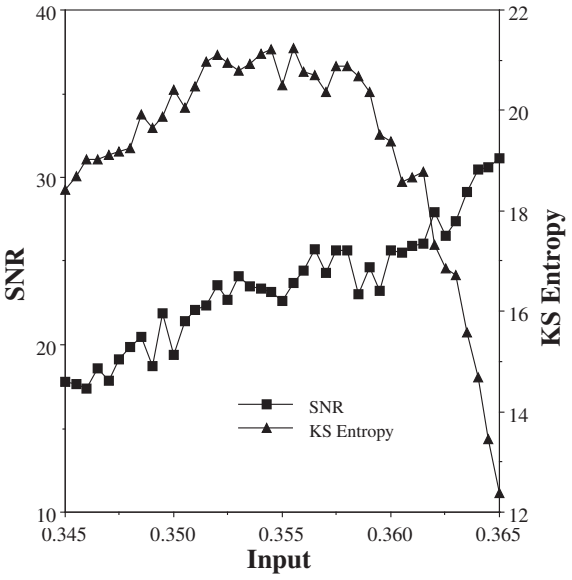


Fig. 4 SNR of mean field $m(t)$ and corresponding KS entropy as a function of input a when weight $w = 0.0055$ in a global coupling network.

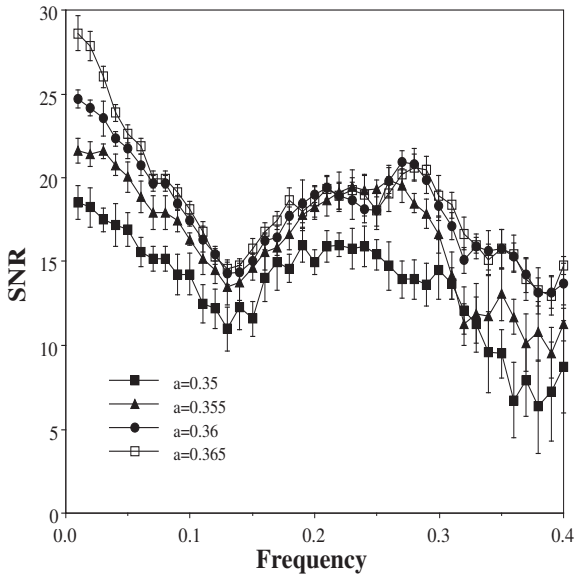


Fig. 5 SNR of mean field $m(t)$ as a function of drive frequency f_m in a global coupling network when weight $w = 0.005$.

level to promote the SNR. The SNR values of a mean field at $w = 0$ are superior to the SNR values of the driven single neuron model ($w = 0$) because of the law of large numbers. This enhancement by summing alone is as promised.

5. If we compare the SNR values of a neuron in $w > 0$ (global coupling network) and those at $w = 0$ (single neuron model), the parameter regions are such that the SNR values of a neuron in $w > 0$ are superior to those of a neuron at $w = 0$. There is enhancement on a neuron level in a coupling network, but it is not always enhanced as in the mean field. The enhanced areas are much larger than those of the SNR of a mean field.

To show the relation between the SNR of a mean field and the KS entropy, we represent a set of plots of the SNR and KS entropy in Fig. 4. We can find parameter regions in larger values of the weight and input that have a negative correlation between the SNR of a mean field and the KS entropy. This negative correlation also appears in smaller values of weight as well as in the results of the driven single neuron model [24]. However, other parameter regions have no clear correlation between the SNR and KS entropy because of lack of marked value changes. This situation is similar to the correlation between the SNR of a neuron and the KS entropy.

Next, we also consider the SNR dependence on the drive frequency and amplitude. We calculated the SNR value when weight $w = 0.005$ by averaging 10 initial value sets. Figure 5 shows the SNR of mean field $m(t)$ depending on drive frequency f_m. There are regions of SNR enhancement around low frequencies below $f_m = 0.1$ and $f_m = [0.2, 0.3]$. The regions of resonance frequencies are clearer than in the driven single neuron model [24]. However, they are unclear compared to general resonance phenomena. There are smoother curves for the dependence on frequency in Fig. 5 than in plots for the dependence in the driven single neuron model. Figure 6 shows the SNR of mean field $m(t)$ depending on drive amplitude A. Increasing the drive amplitude saturates the SNR. Further increases in the amplitude make the driven system nonchaotic. These situations are similar to the results of the driven single neuron model [24].

Finally, we examine the SNR on the nearest neighbor coupling network with a periodic or free boundary. Figure 7 shows the dependence of the SNR for one of neuron internal states $y_i(t)$ and the corresponding KS entropy on weight w and input a in a network with a periodic boundary. We represent Fig. 8 to clarify the relations among the SNR values and those between the SNR and KS entropy. The parameter regions in larger values of the weight and input are such that the SNR values of a neuron in $w > 0$ (nearest neighbor coupling network) are superior to those of a neuron at $w = 0$ (single neuron model). There is enhancement on a neuron level, but it is not always enhanced as in the results of the global coupling network. We find parameter regions in smaller and larger values of the weight that have a negative correlation between the SNR and KS entropy. However, other parameter regions have no clear correlation.

Figure 9 shows the dependence of the SNR for one of

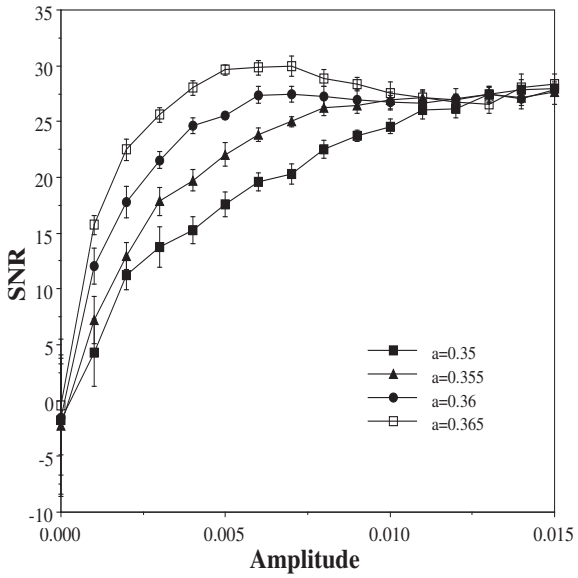
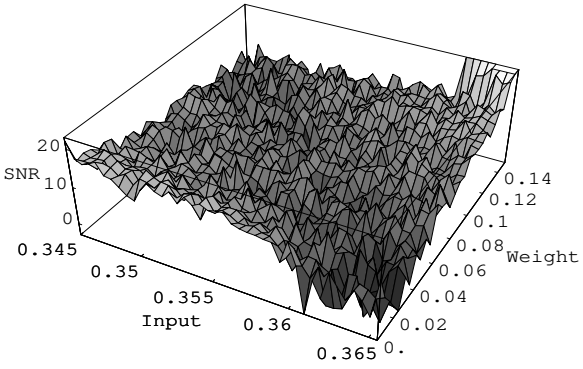
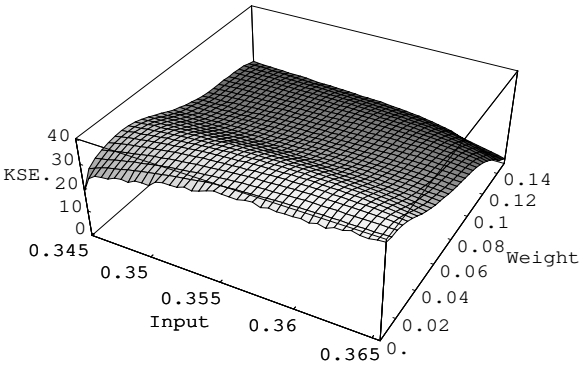


Fig. 6 SNR of mean field $m(t)$ as a function of drive amplitude A in a global coupling network when weight $w = 0.005$.



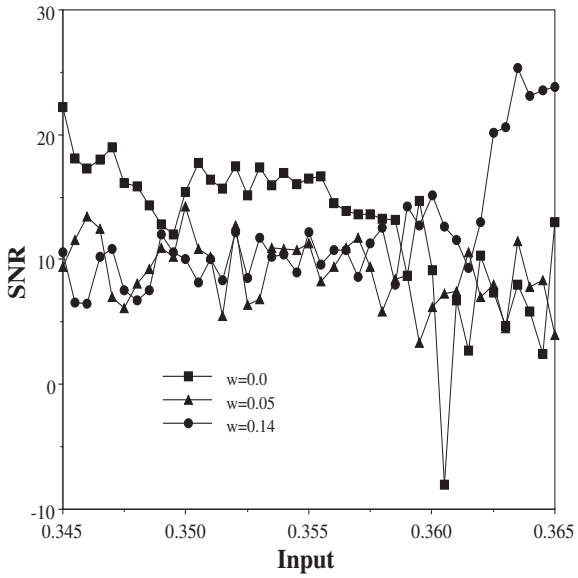
(a) SNR of one of neuron internal states $y_i(t)$.



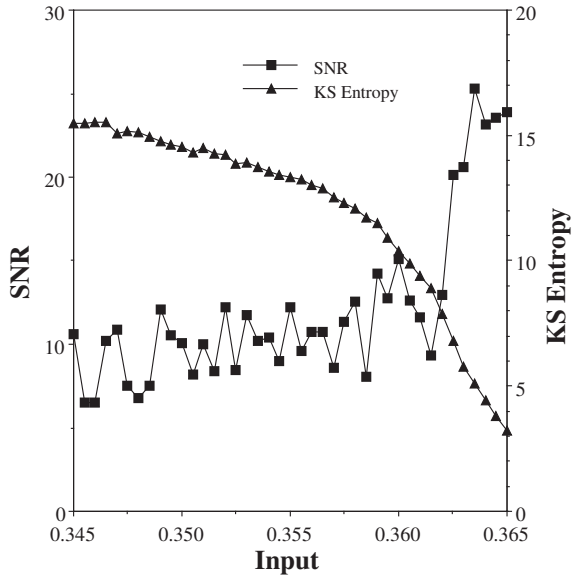
(b) Corresponding KS entropy.

Fig. 7 SNR of one of neuron internal states $y_i(t)$ and corresponding KS entropy as a function of weight w and input a in the nearest neighbor coupling network with a periodic boundary.

neuron internal states $y_i(t)$ and the corresponding KS



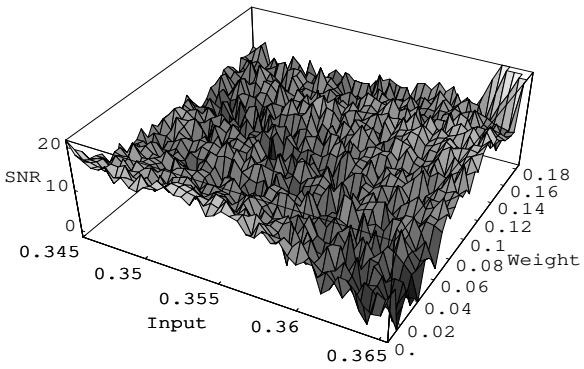
(a) SNR of a neuron $y_i(t)$.



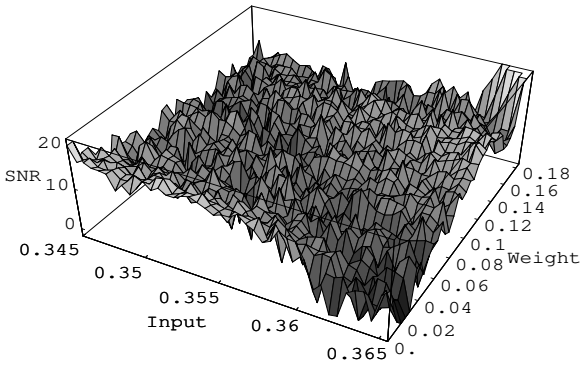
(b) SNR of a neuron $y_i(t)$ and corresponding KS entropy when weight $w = 0.14$.

Fig. 8 SNR of a neuron $y_i(t)$ and corresponding KS entropy as a function of input a at fixed weight values in the nearest neighbor coupling network with a periodic boundary.

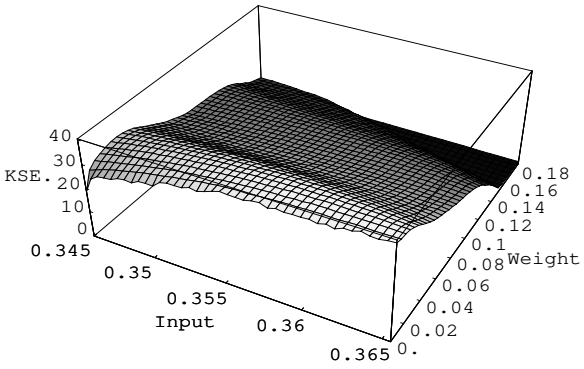
entropy on weight w and input a in a network with a free boundary. In Fig. 10, there are relations among the SNR values similar to the results with a periodic boundary. We also find parameter regions in smaller and larger values of the weight that have a negative correlation between the SNR and KS entropy. However, other parameter regions have no clear correlation. Figure 11 shows the dependence of the SNR on the neuron's positions in the nearest neighbor coupling network



(a) SNR of neuron internal state $y_{10}(t)$.

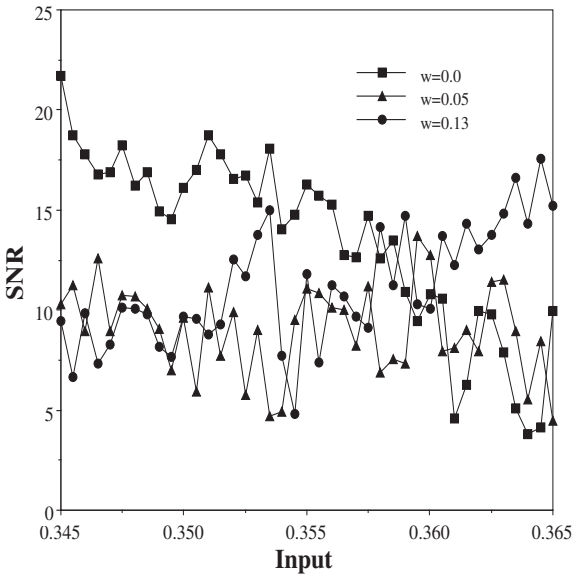


(b) SNR of neuron internal state $y_{50}(t)$.

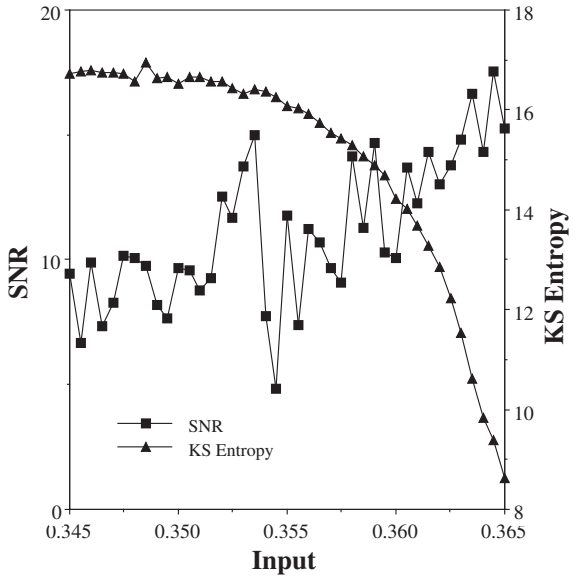


(c) Corresponding KS entropy.

Fig. 9 SNR of one of neuron internal states $y_i(t)$ and corresponding KS entropy as a function of weight w and input a in the nearest neighbor coupling network with a free boundary.



(a) SNR of a neuron $y_{10}(t)$.



(b) SNR of a neuron $y_{10}(t)$ and corresponding KS entropy when weight $w = 0.13$.

Fig. 10 SNR of a neuron $y_{10}(t)$ and corresponding KS entropy as a function of input a at fixed weight values in the nearest neighbor coupling network with a free boundary.

with a free boundary when input $a = 0.35$. The difference of the SNR value depends on a chaotic fluctuation, not on a neuron's position from the edge.

5. Conclusions and Discussions

We numerically calculated properties of resonance in chaotic neural networks driven by a weak sinusoid in the turbulent phase. Summing networks enhanced the SNR of a mean field when the neuron number in the network increased. The enhanced mechanism is understood by the law of large numbers. The global coupling enhanced each SNR of a mean field and a neuron depending on the parameters, but the coupling did not always enhance them. Summing also enhanced the SNR in the coupling network. A combination of coupling and summing enhanced the SNR, but summing that provides a mean field was more effective than coupling on a neuron level to promote the SNR. Only summing

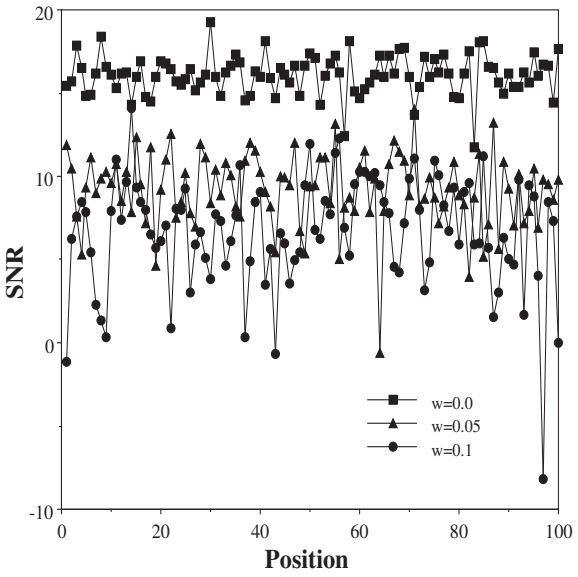


Fig. 11 SNR of neuron internal states $y_i(t), i = 1, \ldots, N$ as a function of position i in the nearest neighbor coupling network with a free boundary when input $a = 0.35$.

assured the enhancement based on the law of large numbers. There were parameter regions with a negative correlation between the SNR and KS entropy both in a mean field and a neuron. The driven single neuron model [24] has similar results in the network. The SNR in the network depends on not only the KS entropy but also the phase structure of chaotic attractors. The SNR also depends on changes in the phase structure of a chaotic attractor when the bifurcation parameter is slowly modulated by a sinusoid. The KS entropy is related to the phase structure of a chaotic attractor, but does not have a one-to-one correspondence to the exact phase structure. We can not explain the negative correlation between the SNR and KS entropy from the limited facts and analyses. Therefore, more data are necessary for a qualitative interpretation.

We found SNR enhancement around a few frequencies. These resonated frequencies were clearer than in the driven single neuron model. We also noted the significantly smoother dependence on frequency compared to the driven single neuron model. An increase in the drive amplitude saturated the SNR, and further increases made the driven system nonchaotic. These dependencies on the drive frequency and amplitude showed similarities to the results of the driven single neuron model.

The nearest neighbor coupling network with a periodic or free boundary also enhanced the SNR of a neuron, but the coupling did not always enhance. There was also a negative correlation between the SNR of a neuron and the KS entropy. In the network with a free boundary, there was a tendency that the effect on the SNR from both edges of the free boundaries is small.

Finally, we mention that it is interesting to study the spatiotemporal pattern of resonance in an inhomogeneous coupling network as an associative memory network. We have studied a perturbed system of an associative memory based on a chaotic neural network, which shows the chaotic itinerancy around the memorized patterns [37].

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