PAPER

Resonance in a Chaotic Neuron Model Driven by a Weak Sinusoid

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SUMMARY We show by numerical calculations that a chaotic neuron model driven by a weak sinusoid has resonance. This resonance phenomenon has a peak at a drive frequency similar to that of noise-induced stochastic resonance (SR). This neuron model was proposed from biological studies and shows a chaotic response when a parameter is varied. SR is a noise induced effect in driven nonlinear dynamical systems. The basic SR mechanism can be understood through synchronization and resonance in a bistable system driven by a subthreshold sinusoid plus noise. Therefore, background noise can boost a weak signal using SR. This effect is found in biological sensory neurons and obviously has some useful sensory function. The signal-to-noise ratio (SNR) of the driven chaotic neuron model is improved depending on the drive frequency; especially at low frequencies, the SNR is remarkably promoted. The resonance mechanism in the model is different from the noise-induced SR mechanism. This paper considers the mechanism and proposes possible explanations. Also, the meaning of chaos in biological systems based on the resonance phenomenon is considered.
key words: chaotic neuron model, biological information processing, driven chaotic system, stochastic resonance, numerical calculation

1. Introduction

Background noise with respect to a signal is considered a nuisance. Engineers have always striven to reduce background noise, but in some nonlinear systems such as a bistable system, it yields the benefit of weak signal detection. This phenomenon is called stochastic resonance (SR) [1]–[3]. The basic SR mechanism is that the system is synchronized and resonated by an external periodic signal when a bistable system is driven by a subthreshold sinusoid plus some amount of noise. The subthreshold sinusoid alone can not overcome the potential barrier of the bistable system, but the addition of noise helps this movement between wells of bistable potentials. This timing is occasionally synchronized to an external periodic signal. The power spectrum of the system's time series has peaks at the drive-signal frequency and its harmonics. Suppose that a detector can catch the signal when the system's state overcomes the potential barrier. The power spectrum of the external signal, which contains the sinusoid and noise has no peak at the sinusoid frequency. In a SR situation, the subthreshold sinusoid can be found as a peak with background noise in the power spectrum of a detector's response. This indicates that SR enhances the detection of a weak signal from background noise. SR is also observed in threshold systems, but the result is a noise-induced threshold crossing rather than resonance, because the system is independent of the drive signal frequency [4], [5]. Excitable media also show SR [6]. For these reasons, it has been shown that neuron models such as the Fitzhugh-Nagumo model [7], [8], exhibit SR behaviors [9]. This SR effect is found in the actual sensory neurons of crayfish and obviously provides a useful sensory function for the segmentation of noisy backgrounds [10].

SR is known as an effect driven by two cooperative signals, i.e., a sinusoid and noise [1]–[3]. This situation considered as a perturbed system is more complex than that for one with a driving-signal case. In this paper, we consider a chaotic system as a fluctuating system instead of a bistable system driven by noise. There has been a lot of studies on nonlinear systems perturbed by adding a signal or by multiplying a parameter by a signal. However, few papers have addressed perturbed chaotic systems in the context of SR. Crisanti et al. [11] mainly considered the SR effect in a noiseless deterministic chaotic system when the bifurcation parameter was driven periodically in time below and above the threshold for the onset of chaos. The mechanism of this resonance is the same as in an excitable system driven near a bifurcation point.

Anishchenko et al. considered noise-induced and noiseless SR in the cubic map [12] and the Chua's circuit [13]. The cubic map and the Chua's circuit have a bifurcation which a chaotic attractor separates into two attractors. This has been studied as a phenomenon of intermittency of the chaos-chaos type [12], [14]. In the noise-induced SR case, the noise can merge the basins of the individual chaotic attractors. The basic resonance mechanism in this case is chaos-chaos switching synchronized to an external sinusoid. Even in the noiseless case, resonance was found having a peak in the power spectrum. This mechanism was modulated chaos-chaos intermittency synchronized to an external

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sinusoid. SR can be found in bistable systems of not only fixed points but also any kind of attractor, even chaos. In noise-induced SR, the noise can merge the basins of the individual chaotic attractors.

Nicolis et al. [15] studied SR-like enhancement in the bimodal chaos of the Duffing equation and a one-dimensional intermittent map driven by a weak sinusoid alone. The Duffing equation is a nonautonomous system driven by a primary sinusoid and generates a chaotic attractor reflected by aperiodic jumps between two stable fixed points $x = \pm 1$ of an unperturbed system. They subjected it to a secondary periodic forcing in the form of an additive contribution. The amplitude of the secondary periodic forcing was much less than that of the primary sinusoid. In addition, they studied how the bimodal chaos of the Duffing equation shows SR-like enhancement by driving by a weak secondary sinusoid, and mainly calculated the probability distribution of residence times in one part of the phase space (i.e., $x \leqq 0$ or $x > 0$). We see a sequence of discrete peaks instead of a smooth exponential-like distribution in the probability distribution. The probability mass of the principal peak is depressed by the secondary forcing and the probability mass of peaks of higher periods is enhanced when the period is a certain multiple of the period of the primary sinusoid. The secondary forcing drifts the mean residence time to a higher value. Nicolis et al., however, reported that any obvious peak in the power spectrum and a high SNR value can not be identified. By the above results, they showed an enhanced sensitivity to external periodic signals, such as chaos spontaneously generates its own effective noise in this system.

We also do not consider any external stochastic forces in this paper, but similar to previous research, we regard a chaotic fluctuation as stochastic noise in the studied system. Note that the ratio of an input signal to an input noise for these systems is infinite because the external input is a noise-free sinusoid. In this paper, according to previous studies, we attach great importance to understanding the resonance properties and its mechanism, not to detect a weak signal as a noise-induced effect. Stochastic noise is generated intrinsically from the system as chaos. If we could consider intrinsic spontaneous noise as deterministic chaos, the nerve system in a biological system may benefit from the resonance generated in the driven chaotic system. Accordingly, we also emphasize our consideration on how an autonomous chaotic system shows a resonance phenomenon by driving by a weak sinusoid. Nonautonomous systems such as the Duffing system are necessary with a sinusoid even to produce chaos.

Here, we consider a chaotic neuron model as a biological chaotic system. The chaotic neuron model and its network were proposed by Aihara et al. and are based on their studies of squid giant axons and the Hodgkin-Huxley equation [16]. The networks of the neuron model have been intensively studied in information processing, with consideration on escaping from local minima in optimization and retrieving an associative memory [17]–[19]. Recently, some chaos control techniques have also been applied to networks to stabilize the chaotic retrieval processes [20], [21]. The neuron model is a generalization of the McCulloch-Pitts model [22] as a threshold system, or the Fitzhugh-Nagumo model [7], [8] as an excitable system. This model has also become a chaotic system. This was the first simple neuron model to provide a chaotic response; some parameter combinations trigger a chaotic output. The driven model does not show chaos-chaos intermittency because the model does not simultaneously have two chaotic attractors in a certain parameter set.

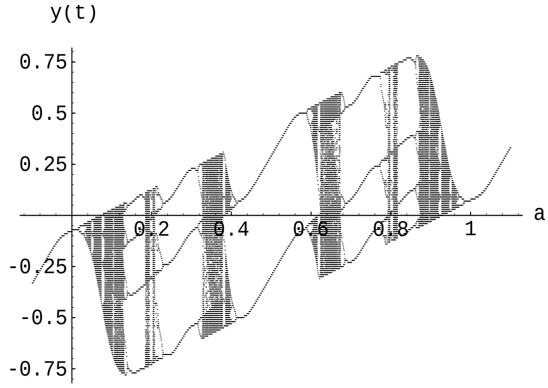
In our study on the resonance phenomenon in the chaotic neuron model, we first show that, similar to the SR in the bistable case, the chaotic neuron model driven by a weak sinusoid also has obvious peaks in the power spectrum. The resonance mechanism is different from noise-driven SR because of the lack of chaotic multi-attractors. Next, we show the dependence of the signal-to-noise ratio (SNR) on the model parameter and the drive parameters. Finally, we draw some conclusions from the results and discuss the resonance mechanism.

2. Chaotic Neuron Model

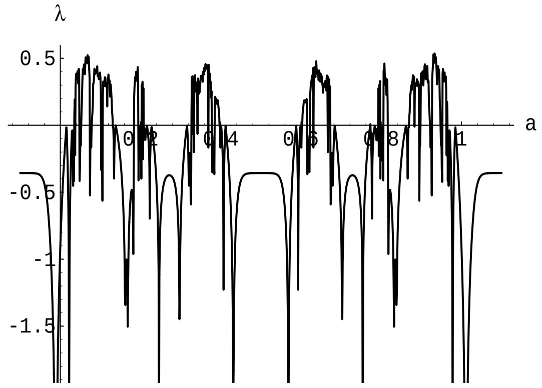
The original chaotic neuron model is represented as the dynamics of the difference equations of three variables [16]. A certain parameter combination can reduce these three variables into one variable. The simplest chaotic neuron model has the following dynamics.

$$\begin{aligned} y(t+1) &= ky(t) - \alpha f(y(t)) + a, \\ x(t+1) &= f(y(t+1)), \\ f(z) &= \frac{1}{1+\exp(-\frac{z}{\epsilon})}, \end{aligned} \tag{1}$$

where
$y(t)$: internal state of neuron at time t,
$x(t)$: output of neuron at time t,
k : damping constant,
α : refractory parameter ($\alpha \geqq 0$),
a : parameter based on the input and the threshold,
ϵ : steepness parameter of the sigmoid function.
We investigate this model for simplicity. The neuron has a chaotic output when parameter a yields a positive Lyapunov exponent. The Lyapunov exponent characterizes how arbitrarily a small disturbance is exponentially amplified or contrasted. A bifurcation diagram and the Lyapunov exponent as a function of parameter a are shown in Fig. 1. We fix the parameter set at $k = 0.7$, $\alpha = 1.0$, and $\epsilon = 0.02$ throughout this paper.



(a) Bifurcation diagram



(b) Lyapunov exponent

Fig. 1 Bifurcation diagram and Lyapunov exponent of a chaotic neuron as a function of input parameter a. The parameters are $k = 0.7$, $\alpha = 1.0$, and $\epsilon = 0.02$. In Fig. 1(b), the line connecting the points is only a guide for the eyes.

3. Fundamental Mechanism of Stochastic Resonance

SR is a noise-induced effect in nonlinear systems [1]–[3]. Consider an overdamped ball $x(t)$ moving in double-well potential $U(x)$ such that

$$\frac{dx}{dt} = -\frac{\partial U}{\partial x} + F(t) + A \sin 2\pi f_0 t \tag{2}$$

with a weak time-periodic signal $A \sin 2\pi f_0 t$ and noise $F(t)$ (Fig. 2(a)). The ball is weakly forced sinusoidally in time, but can not overcome potential barrier ΔU with a weak periodic signal alone. A certain amount of force is required for the ball to overcome the barrier. If we add noisy fluctuating force $F(t)$, the ball can occasionally jump over the barrier. Here, $F(t)$ is Gaussian white noise $[\langle F(t)\rangle = 0, \langle F(t)F(t')\rangle = 2D\delta(t-t')]$. The chance of switching the wells is synchronized with the amplitude peak of the periodic signal. Time series $x(t)$ of the incidents, where the barrier is overcome, has peak $S(f_0)$ at sinusoid frequency f_0 in the power spectrum of Fig. 2(b). This peak does not appear when the noise is excessively small or excessively large as in Fig. 2(c). This is because in the former case the incidents (where the barrier is overcome) do not occur, and in the latter they occur at random. There is an optimum amount of noise to detect the weak signal for any given SNR in Fig. 2(c) [for example, SNR= $10 \log_{10}(S(f_0)/N)$]. This phenomenon is called SR.

When we consider the detection of the weak sinusoid in this bistable system, we can find an intriguing characteristic. More specifically, we can not detect the weak sinusoid from the power spectrum of the original signal, which contains the sinusoid plus noise, because the power of the sinusoid is covered by the noise power. Suppose that the system's movements are detectable only when the system's state jumps from one well to the other. If the sinusoid alone is too weak under a threshold of a detector, the detector will not find it. Once the fluctuation is introduced to the system in a moderate amount, the detector responds occasionally because the total amount of signals occasionally exceeds the threshold. To detect the sinusoid, we can statistically process the output from the detector to extract the power peak. Background noise can enhance the detection of a weak time-periodic signal.

The mechanism of stochastic resonance was originally proposed as an explanation of the periodic occurrences of ice ages on earth [23]. A weak periodic shift of earth's orbit to the sun is suspected as a recurrence of glaciation every $100,000$ years. However, this orbital eccentricity alone can not present a major reason for the energy difference between earth's climates. To solve this difficulty, a group of scientists described the concept of driven nonlinear dynamics, where a small periodic perturbation can be amplified by fluctuations [23]. They assumed that the climates of earth have two stable states as a bistable system. A periodic orbital shift alone can not cause a switch in climates, but adding orbital fluctuations can overcome the potential difference between the bistable states, and the timing of glaciation synchronizes to the periodicity of the orbital eccentricity. The discussion still continues, but it is an interesting model. Note that not only this case, but also situations in which a signal is amplified by fluctuations, have been found in many systems.

SR has been reported in a wide range of physical systems, not only bistable systems but also excitable systems and threshold systems. As such a system, we can consider a neuron model with integrate-and-fire dynamics. Dynamics of this type can be described in two parts. The first is "integration", i.e., when the integrated input to a neuron exceeds a certain threshold, and the second is "fire", because the neuron fires and resets the integration. A simplified version of a neuron model known as a threshold system is called the McCulloch-Pitts model [22].

SR has also been investigated even in actual neurons, e.g., a mechanoreceptive sensory neuron from crayfish [10], and from rat skin [24], and an interneuron from the cricket abdominal ganglia [25]. The neural network

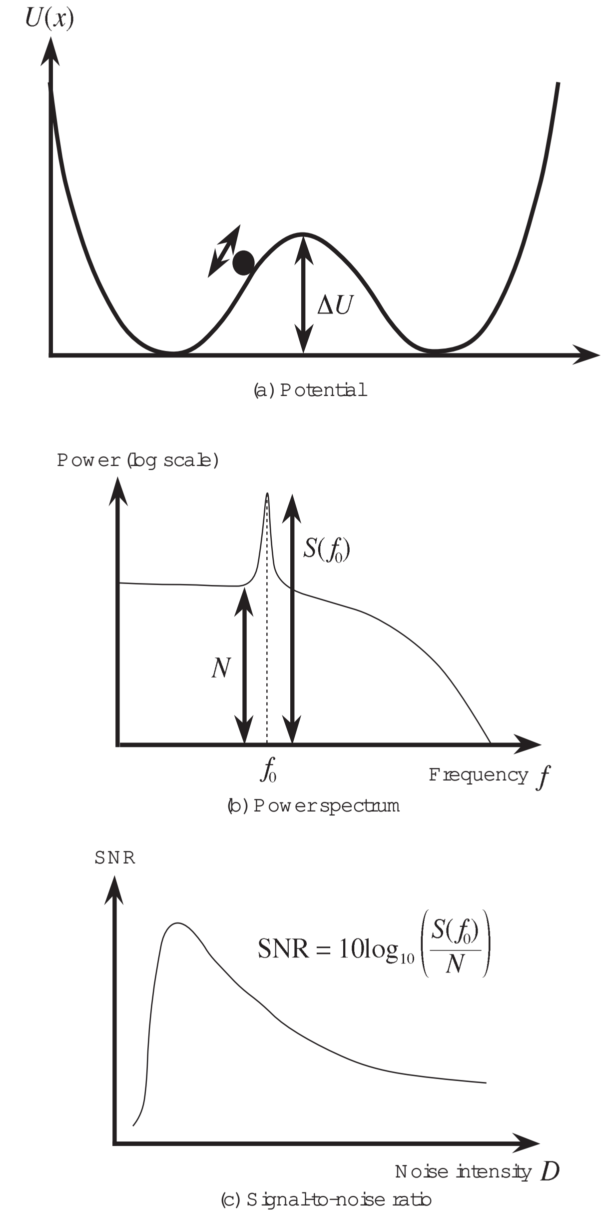


Fig. 2 Stochastic resonance

of a biological system has been investigated with respect to SR. The extracellular potential of hippocampal slices in a rat temporal lobe showed SR in the electric fields of a sinusoid plus noise [26]. The mechanism has also been introduced to explain fluctuations in the human visual perception in resolving ambiguous figures [27], for example, known as the Necker cube [28] in visual psychology. In biological systems, studies on SR are considered to give an understanding of the sensory mechanism in real noisy environments and high-level cognitive processes. A technological application using SR has been intensively investigated in superconducting quantum interference devices (SQUIDs) [29]. This application aimed at the robustness of the SQUIDs' operation in a fluctuating magnetic field.

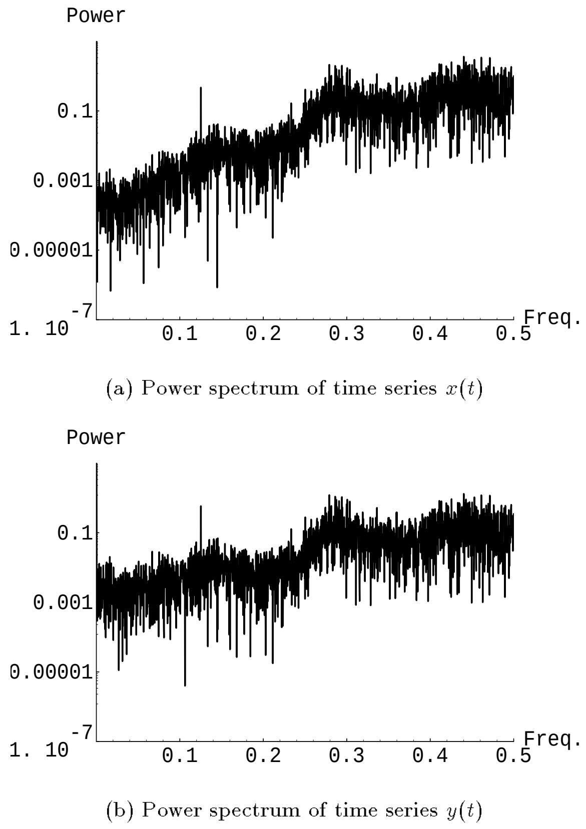


Fig. 3 Power spectra of the chaotic neuron model driven by a weak sinusoid.

4. Resonance in the Chaotic Neuron Model

We investigated the following chaotic neuron model driven by a sinusoid with weak amplitude A and frequency f_m.

$$\begin{aligned} y(t+1) &= ky(t) - \alpha f(y(t)) + a + A\sin 2\pi f_m t, \\ x(t+1) &= f(y(t+1)). \end{aligned} \tag{3}$$

We used the parameters $a = 0.35$, $A = 0.005$, and $f_m = 0.125$ and time step $T_n = 4096$ to calculate the power spectrum, unless otherwise mentioned. These values make the model chaotic. Figure 3 shows typical power spectra of time series $x(t)$, $y(t)$ obtained by discarding 8192 transient time steps. Both spectra have sharp peaks at a drive frequency f_m of 0.125. Higher harmonics can be observed as the signal amplitude increases. For the sigmoid function, similar to SR effect of threshold systems, the power spectrum of $x(t)$ has a higher peak than that of $y(t)$. Accordingly, we investigated the resonance of $y(t)$ rather than $x(t)$ to study the chaotic neuron model as a nonlinear map without the sigmoid function effect.

We calculated the dependence of the SNR on input parameter a of the model parameters, and amplitude A and frequency f_m of a drive signal. We defined the SNR in time series $y(t)$ as

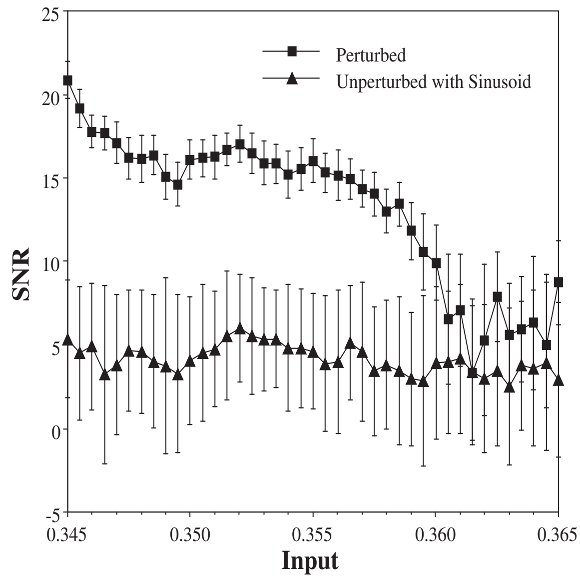
$$\mathrm{SNR} = 10\log_{10}(\frac{S(f_m)}{N}). \quad (4)$$

Here, $S(f_m)$ is the peak power at the drive signal frequency and N is the noise power obtained by averaging 40 frequency points around the signal frequency. We calculated the SNR average from 100 chaotic time series that were acquired on 100 different initial values of random numbers of $y(0) = [0, 1]$. The parameter dependence of the SNR was examined by successively varying the value of one parameter while keeping the values of the other two fixed at the above values. The Lyapunov exponent was calculated from 8192 time steps discarding 8192 transient time steps.

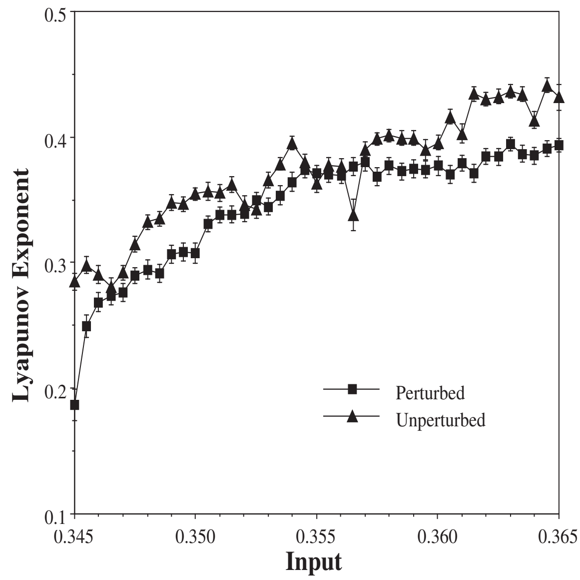
Figure 4 shows the SNR vs input parameter a. We chose the region of $a = [0.345, 0.365]$ where there was an increasing trend in the Lyapunov exponent of the unperturbed model as shown in Fig. 4(b). The perturbed system offers an improved SNR compared to the unperturbed time series with the addition of a weak sinusoid. There is a negative correlation between the SNR value and the Lyapunov exponent in this figure. However, based on calculations in other parameter regions, the SNR depends on not only the Lyapunov exponent but also the phase structure of chaotic attractors. The SNR property does not converge to a smooth line if the number of samples increases when calculating the average SNR, because the chaos in the map is structurally unstable.

Figure 5 shows the SNR vs amplitude A of the drive signal. Here, like the above case, the perturbed system offers a higher SNR than the unperturbed time series with a weak sinusoid. The SNR value becomes saturated with the magnitude of fluctuations as the drive amplitude increases. However, the perturbed system becomes nonchaotic as the drive amplitude increases further.

Figure 6 shows the SNR vs frequency f_m of the drive signal. The perturbation improves the SNR in subsequent frequency regions, at low frequencies $f_m = [0.05, 0.25]$ and around $f_m = 0.3$. An improved SNR value in this region can also be found when the drive frequency is an irrational number. Obviously, this model contains some mechanism in which a given weak drive signal makes the SNR improve. The highest SNR peak by far is at a low frequency, i.e., where the Lyapunov exponent is very low. This high SNR point is singular because no other such high SNR point can be found from fine point calculations around that point.



(a) SNR vs input parameter a

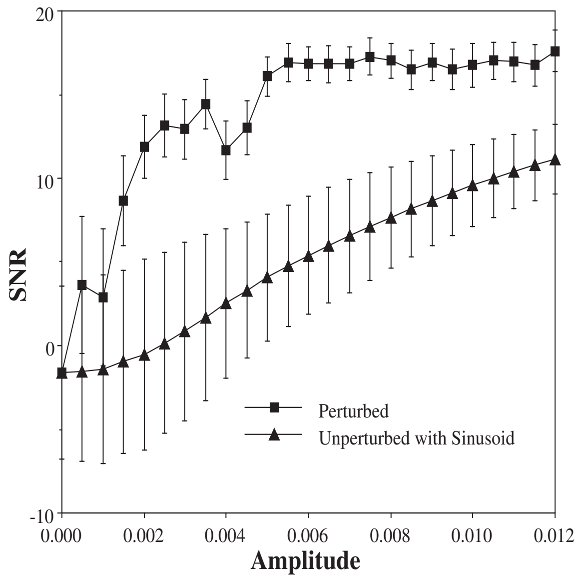


(b) Lyapunov exponent vs input parameter a

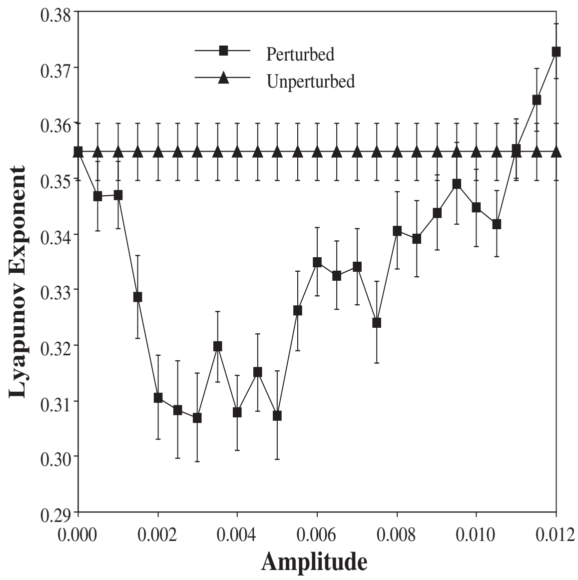
Fig. 4 SNR and corresponding Lyapunov exponent as a function of input parameter $a = [0.345, 0.365]$. In Fig. 4(a), the square plots represent the SNR for a perturbed time series and the triangle plots represent the SNR for an unperturbed time series with the addition of a weak sinusoid. In Fig. 4(b), the square plots represent the Lyapunov exponent for a perturbed time series and the triangle plots represent the Lyapunov exponent for an unperturbed time series (no weak sinusoid is added). All of the lines connecting points are guides for the eyes.

5. Conclusions and Discussions

We investigated a chaotic neuron model driven by a weak sinusoid using numerical calculations. Our results revealed noise driven SR-like behaviors with a power

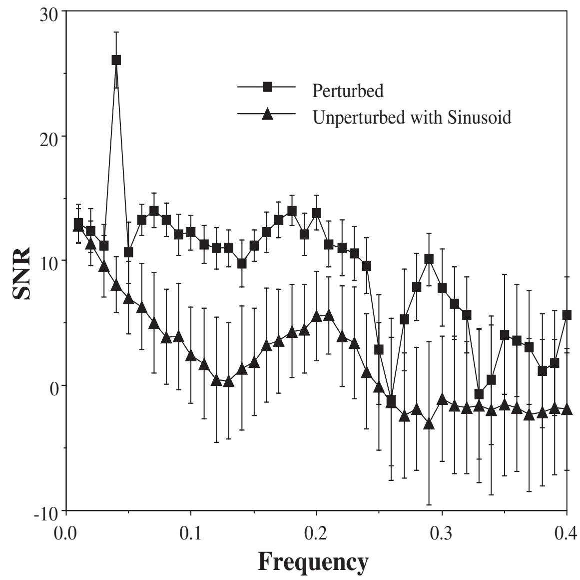


(a) SNR vs drive amplitude A

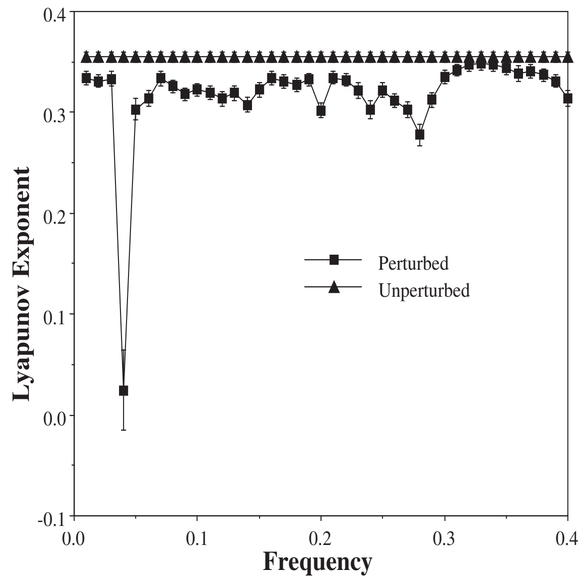


(b) Lyapunov exponent vs drive amplitude A

Fig. 5 SNR and corresponding Lyapunov exponent as a function of drive amplitude A. The lines here follow those in Fig. 4.



(a) SNR vs drive frequency f_m



(b) Lyapunov exponent vs drive frequency f_m

Fig. 6 SNR and corresponding Lyapunov exponent as a function of drive frequency f_m. The lines here follow those in Fig. 4.

spectrum peak at a drive frequency. We calculated the SNR as functions of the input parameter of the model, and the amplitude and frequency of the drive signal. First, depending on the input parameter, the SNR of a perturbed time series was improved compared to an unperturbed time series with a weak sinusoid. The SNR depended on both the Lyapunov exponent and a phase structure of chaotic attractors. We could not merely regard the Lyapunov exponent as the intensity of noise in noise-induced SR. However, there was a negative correlation between the SNR and the Lyapunov exponent. Next, depending on the drive amplitude, the SNR of the perturbed system was also improved when the amplitude increased. The SNR value was saturated with increasing drive amplitude, but the perturbed system became nonchaotic with a further increase in the drive amplitude. Finally, depending on the drive frequency, the SNR of the perturbed system was improved considerably mainly at low frequencies. Even from this result, this neuron model must have some resonance mechanism that makes the SNR larger by a given weak drive signal.

The mechanism of this phenomenon is described. First, the possibility that the approximated system may have a bistable potential is brought to light. The Hopfield-type neuron model has a bistable potential in a certain case [30]. The chaotic neuron model, in contrast, does not have a bistable potential similar to the Hopfield-type model even if the model with a given parameter set is approximated by a differential equation $[dx/dt = x(t+1) - x(t)]$. The SNR of the approximated system can not be estimated by an adiabatic approach [31]. Second, the mechanism may be explained as noise induced threshold crossing; the equation of the model includes some effect from a sigmoid function similar to that of threshold systems. The SNR of threshold systems is independent of the drive frequency, but the SNR of the model is frequency-dependent. Third, and more interesting, the perturbation can be regarded as a slow change in bifurcation parameter a (input parameter). When the phase structure of an attractor is sensitive to this parameter, the slow perturbation - slow compared to the time scale of the attractor - derives a resonance phenomenon. The stochastic system generating noise-induced SR has its own selectivity of response to an external periodic signal and amplification of the signal. The response is mainly to the slow periodic perturbation and even aperiodic [32]; the frequency is slower than the characteristic time of the system [31]. The response to a low frequency of a periodic signal is also characteristic of the chaotic neuron model. The slow weak perturbation is the origin of the resonance and high SNR in the driven model; being slower than the time scale of the chaotic attractor is essential.

We can check other chaotic maps parametrically driven by a sinusoid to adopt the third possibility as the mechanism. If it is the mechanism, we can find an improved SNR when the perturbation is slow enough compared to the time scale of chaotic fluctuations. In other maps, we can observe a power spectrum peak and corresponding high SNR that is dependent on the drive frequency [33]. However, some of the maps have scattered frequencies of an improved SNR, and others do not show any significant improvement in the SNR. One of them shows an improved SNR of harmonics frequencies. In this case, the power of the drive frequency moves to the harmonics frequencies by a nonlinear effect. These facts support the finding that the mechanism may be related to not only the third possible scenario but others that we described and several kinds of nonlinear map functions. Actually, biological systems may use such a mechanism by chaos to process information. We can see that a driven system can improve the SNR when the state of the system is in weak chaos rather than in strong chaos. Assuming that spontaneous noise in biological systems can be regarded as deterministic chaos, we can say that biological systems may take advantage of chaos-generated phenomena. An example of this is resonance generated by chaos as it becomes effective noise in a spontaneous manner.

From studies on nonlinear parametric effects, we are able to understand that the bifurcation point is shifted by periodic perturbations [34], [35]. There are some reports stating that some chaotic dynamics are suppressed by perturbations. Lima and Pettini analyzed the suppression of chaos in the Duffing-Holmes equation by resonant parametric perturbations [36]. Braiman and Goldhirsh reported taming chaos in an equation of a Josephson junction by weak sinusoid perturbations [37]. These results may give us some clues to understanding the resonance mechanism.

Finally, several studies on enhanced resonance in extended systems have recently been reported [32], [38], [39]. We mention that it would be very interesting to investigate resonance in a network of the chaotic neuron model. A study on the properties of network resonance and a comparison of the neuron resonance may improve our understanding of the resonance mechanism, because we would be able to check the third possibility of the mechanism in this paper indirectly from differences in the properties of chaos from the single map and the coupled map. We may obtain a smooth property for changes in the SNR value against the bifurcation parameter and drive parameters because high dimensional coupled systems have structural stable chaos [40] in chaos regimes. We can also find a clearer resonance frequency from a smooth SNR property reflected in the stability.

Acknowledgments

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