DECORRELATION-BASED BLIND SOURCE SEPARATION ALGORITHM AND CONVERGENCE ANALYSIS

Tiemin Mei, Fuliang Yin

School of Electronic and Information Engineering, Dalian University of Technology, Dalian, Liaoning, P. R. China 116023, (E-mail: meitiemin@163.com).

ABSTRACT

The algorithm for the blind separation of two instantaneous mixed sources is presented based on decorrelation. Its convergence properties are analyzed qualitatively and quantitatively. The algorithm is suitable for both stationary and nonstationary, super-Gaussian and sub-Gaussian signal separation. It has the advantages of low computation complexity, fast convergence, independence on the probability distribution, noise robustness and numerical stability. Numerical experiments are presented to illustrate the validity of our algorithm.

Key words: blind source separation, decorrelation, convergence analysis.

1. INTRODUCTION

Blind source separation (BSS) is to separate source signals from observations without the aid of any a priori knowledge about the sources or the mixing system. The only assumption is that the sources are statistically independent.

Many algorithms of BSS are based on nonlinear neural networks, or utilize the high order statistics directly [1-4]. But the algorithms based on second order statistics (SOS) have attracted many researchers too, such as L. Molgedey *et al* [5], K. Matsuoka *et al* [6], A. Belouchrani *et al* [7], C. Chang *et al* [8], S. Choi *et al* [9], *etc*.

Although these algorithms are valid, they are not the simplest for the case of two mixed sources. The convergence speed of these algorithms is seldom analyzed. In this paper, we propose a very simple algorithm for the separation of two instantaneous mixed sources on the basis of decorrelation. Its convergence properties are analyzed qualitatively and quantitatively. The proposed algorithm has some advantages. First, it is very simple in implementation; second, it is independent on the probability distributions of sources; third, it is of high convergence rate; lastly, under some conditions, it is noise robust. Numerical experiments are given also.

2. SEPARATION PRINCIPLE BASED ON DECORRELATION

Assume $S(t)=[s_1(t),s_1(t)]$, where $s_1(t)$ and $s_2(t)$ are two zero mean and uncorrelated source signals, that is,

$$E[s_i(t)]=0, \quad i=1,2 \tag{1}$$

$$E[s_1(t_1)s_2(t_2)]=0, \quad \forall t_1,t_2 \tag{2}$$

Another assumption about the sources is that

$$\begin{vmatrix} r_{s_1s_1}(t_1,t_1') & r_{s_2s_2}(t_1,t_1') \\ r_{s_1s_1}(t_2,t_2') & r_{s_2s_2}(t_2,t_2') \end{vmatrix} \neq 0 \tag{3}$$

for $\forall t_1,t_2,t_1',t_2',t_1\neq t_2$.

Assumption (1) and (2) are the basic hypotheses. Assumption (3) means that the spectra of source signals are not similar to each other when the signals are stationary processes. If the source signals are nonstationary processes, assumption (3) means that the ensemble averaged instantaneous powers of the source signals are not similar to each other.

We assume that $X(t)=[x_1(t),x_2(t)]^T$, where $x_1(t)$ and $x_2(t)$ are the observed signals or named as observations. Without the loss of generality, the relation between observations and sources is as follows

$$X(t)=MS(t) \tag{4}$$

where $M=\begin{bmatrix}1 & a\\ b & 1\end{bmatrix}$ is the mixing matrix, a and b are two unknown mixing parameters.

Let the separation matrix $W=\begin{bmatrix}1 & -w_1\\ -w_2 & 1\end{bmatrix}$.

Denote by $Y(t)=[y_1(t),y_2(t)]^T$ the outputs of separation system. The relation between $Y(t)$ and $S(t)$ is as follows

$$Y(t) = GS(t) \quad (5)$$

where $G = WM$ is the global transform matrix. Expanding (5) in its components, we have

$$y_1(t) = (1 - bw_1)s_1(t) + (a - w_1)s_2(t) \quad (6)$$

$$y_2(t) = (b - w_2)s_1(t) + (1 - aw_2)s_2(t) \quad (7)$$

The crosscorrelation between $y_1(t)$ and $y_2(t)$ is

$$r_{y_1y_2}(t,t') = (1 - bw_1)(b - w_2)r_{s_1s_1}(t,t') + (a - w_1)(1 - aw_2)r_{s_2s_2}(t,t') \quad (8)$$

Under the decorrelation condition, let $t = t_1, t_2$ and $t' = t_1', t_2'$, we have the following group of equations in matrix form, that is

$$\begin{bmatrix} r_{s_1s_1}(t_1,t_1') & r_{s_2s_2}(t_1,t_1') \\ r_{s_1s_1}(t_2,t_2') & r_{s_2s_2}(t_2,t_2') \end{bmatrix} \begin{bmatrix} (1 - bw_1)(b - w_2) \\ (a - w_1)(1 - aw_2) \end{bmatrix} = 0 \quad (9)$$

Taking into account the assumption (3), there must be

$$(1 - bw_1)(b - w_2) = 0 \quad (10)$$

$$(a - w_1)(1 - aw_2) = 0 \quad (11)$$

The above equations (10) and (11) have two solutions. The first is $w_1 = a, w_2 = b$; the second is $w_1 = \frac{1}{a}, w_2 = \frac{1}{b}$. They are the two separation points in separation parameter space (w_1, w_2).

In fact, the two equations in (9) are all hyperbola equations in plane (w_1, w_2). The two solutions are just the two cross-points of these two hyperbolas. In equations (9), (t_1, t_1') and (t_2, t_2') are two arbitrary points in plane (t, t') provided that assumption (3) holds. For stationary processes, assumption (3) requires $(\tau_1 = t_1 - t_1') \neq (\tau_2 = t_2 - t_2')$, that is, points (t_1, t_1') and (t_2, t_2') must be on two different lines paralleling with the line $t = t'$ on the plane (t, t'). But for nonstationary processes, the points (t_1, t_1') and (t_2, t_2') can be any two different points on plane (t, t'). Thus, from the viewpoint of blind source separation, although stationary and nonstationary processes are separable on the basis of decorrelation, the separation of nonstationary processes is easier than that of stationary processes, and there are more freedom of algorithm design for nonstationary processes.

3. ALGORITHMS DEVELOPMENT

For the separation system, the relations between its outputs and inputs are as follows

$$y_1(t) = x_1(t) - w_1x_2(t) \quad (12)$$

$$y_2(t) = x_2(t) - w_2x_1(t) \quad (13)$$

To multiply the equation (12) by $y_2(t')$ and do ensemble average at the two sides, we obtain

$$r_{y_1y_2}(t,t') = r_{x_1y_2}(t,t') - w_1r_{x_2y_2}(t,t') \quad (14)$$

Let $r_{y_1y_2}(t,t') = 0$, $t = t_1$ and $t' = t_1 - \tau_1$, we have

$$w_1 = \frac{r_{x_1y_2}(t_1,\tau_1)}{r_{x_2y_2}(t_1,\tau_1)} \quad (15)$$

To apply the similar procedure to the equation (13), and let $t = t_2$ and $t' = t_2 - \tau_2$, we obtain

$$w_2 = \frac{r_{y_1x_2}(t_2,\tau_2)}{r_{y_1x_1}(t_2,\tau_2)} \quad (16)$$

w_1 and w_2 can be calculated alternatively and iteratively in terms of (15) and (16). After convergence, (w_1, w_2) converges to the separation points.

Generally, we set $t_1 = t_2 = t$; τ_1 and τ_2 are set to be different values. If τ_1 and τ_2 are set to be nonzero, the algorithm is noise robust, because the influence of white noises on the estimation of crosscorrelations will be futile. This shows us the fact that the time-delayed decorrelation has some advantages over the spatial decorrelation (prewhitening). If the sources are nonstationary processes, we can even set $\tau_1 = \tau_2 = 0$ to simplify the algorithm further. In addition, We must constrain w_1 and w_2 over a limited range, such as [-1,1], or the algorithms may fail to converge.

4. CONVERGENCE ANALYSIS

We first discuss the convergence of the algorithm qualitatively, and then analyze the convergence rate quantitatively under an ideal situation.

The curves in Fig.1-4 are calculated according to the following equation.

$$r_{y_1y_2}(t,\tau) = (1 + w_1w_2)r_{x_1x_2}(t,\tau) - w_1r_{x_2x_2}(t,\tau) - w_2r_{x_1x_1}(t,\tau) = 0 \quad (17)$$

where $x_1(t)$ and $x_2(t)$ are the mixtures of two speeches. The mixing parameters are *a*=0.5, *b*=0.6.

Refer to the Fig.1. The curves 1 and 2 are the hyperbolas of $r_{y_1y_2}(t_1,\tau_1) = 0$ and $r_{y_1y_2}(t_2,\tau_2) = 0$

respectively, in terms of the separation parameters (w_1, w_2) in the first iteration. From the initial point $(w_1(0), w_2(0))$, we obtain $w_1(1)$ according to the equation of (15). Point $(w_1(1), w_2(0))$ is corresponding to the point A in Fig. 1; from the point A, we obtain $w_2(1)$ according to the equation of (16), $(w_1(1), w_2(1))$ corresponding to the point B in Fig. 1. Obviously, the point B is nearer to the separation point S(*a*,*b*) than the point A. During the second iteration, we assume that hyperbola $r_{y_1y_2}(t_1+1,\tau_1)=0$ corresponds to the curve 1 too, but hyperbola $r_{y_1y_2}(t_2+1,\tau_2)=0$ corresponds to the curve 4 (under ideal situation, the curve 2 and 4 should be the same for stationary processes). After this period, the point (w_1, w_2) will reach the point D through the point C. Contrarily, if $r_{y_1y_2}(t_2+1,\tau_2)=0$ is not the curve 4 but 3, then $w_2(2)$ will be negative (corresponding to the cross-point of the line CD′ and curve 3). According to the learning rule, we maintain $w_2(2)$ the value of the last time. The rest can be deduced in a similar fashion. The separation parameters (w_1, w_2) will converge to the separation point S(*a*,*b*).

Certainly, there exist the short-time divergence during the iteration, but it dose not affect the separation results seriously. Refer to the Fig. 2. After the *k*th iteration, the parameter point (w_1, w_2) reaches the point A. If during the (*k*+1)th iteration, $r_{y_1y_2}(t_1+1,\tau_1)=0$ corresponds to the curve 2 rather than the curve 3, then $(w_1(k+1) > w_1(k))$. This tendency may maintain some iteration periods, and the (w_1, w_2) diverge from the separation point S(*a*,*b*). But because of the nonstationarities of source signals, hyperbolas $r_{y_1y_2}(t_1+1,\tau_1)=0$ and $r_{y_1y_2}(t_2+1,\tau_2)=0$ alternate their relative positions constantly. Thus the tendency of divergence will disappear in short time.

From Fig. 3, we see that the family of hyperbolas $r_{y_1y_2}(t_1+1,\tau_1)=0$ and $r_{y_1y_2}(t_2+1,\tau_2)=0$ do not go through the same point S(*a*,*b*) precisely, but their cross point are really in a very small domain around the separation point. This can be seen more clearly in Fig. 4. This can not affect the convergence substantially.

In the following, we will analyze the convergence rate under an ideal model. We assume that the sources are stationary processes. The relative positions of hyperbolas $r_{y_1y_2}(t_1,\tau_1)=0$ (curve 1) and $r_{y_1y_2}(t_2,\tau_2)=0$ (curve 2) do not change with the iteration, refer to the Fig. 3. In terms of (9), the slopes of the curves 1 and 2 at the separation point S(*a*,*b*) are

$$K_1 = -\frac{r_{s_2s_2}(\tau_1)}{r_{s_{12}s_1}(\tau_1)} \tag{18}$$

$$K_2 = -\frac{r_{s_2s_2}(\tau_2)}{r_{s_{12}s_1}(\tau_2)} \tag{19}$$

respectively.

According to the Fig. 3, $|K_2| < |K_1|$, that is

$$\left|r_{s_1s_1}(\tau_1)r_{s_2s_2}(\tau_2)\right| < \left|r_{s_1s_1}(\tau_2)r_{s_2s_2}(\tau_1)\right| \tag{20}$$

According to the equations in (9), and from the initial point $w_2(0)=0$, after the first iteration

$$w_1(1) = \frac{1-\left(\frac{b}{aK_1}\right)}{1-\left(\frac{b^2}{K_1}\right)}a \tag{21}$$

where point $(w_1(1), w_2(0))$ corresponds to the point A in Fig. 3.

$$w_2(1) = \frac{1-\left(\frac{K_2}{K_1}\right)}{1-ab\left(\frac{K_2}{K_1}\right)}a \tag{22}$$

where point $(w_1(1), w_2(1))$ corresponds to the point B in Fig. 3. The rest can be deduced in a similar fashion. During the *k*th iteration, $w_1(k)$ and $w_2(k)$ are as follows

$$w_1(k) = \frac{1-\frac{b}{aK_1}\left(\frac{K_2}{K_1}\right)^{p-1}}{1-\frac{b^2}{K_1}\left(\frac{K_2}{K_1}\right)^{p-1}}a \tag{23}$$

$$w_2(k) = \frac{1-\left(\frac{K_2}{K_1}\right)^{q}}{1-ab\left(\frac{K_2}{K_1}\right)^{q}}b \tag{24}$$

where $p = \dfrac{3^{k-1}+1}{2}, q = 3^{k-1}$.

With the proceeding of iteration, p and q

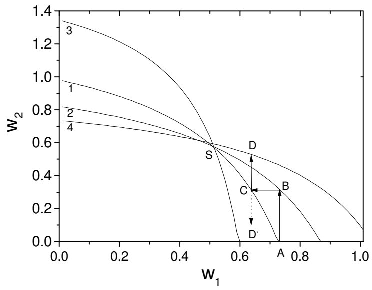

Fig.1. The hyperbola family of w_1 vs w_2 (a)

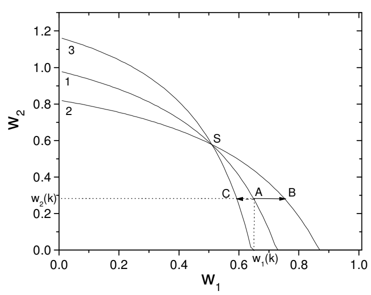

Fig.2. The hyperbola family of w_1 vs w_2 (b)

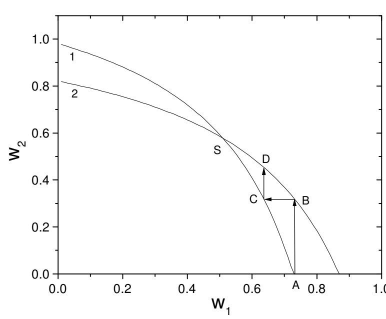

Fig.3. The hyperbola family of w_1 vs w_2 (c)

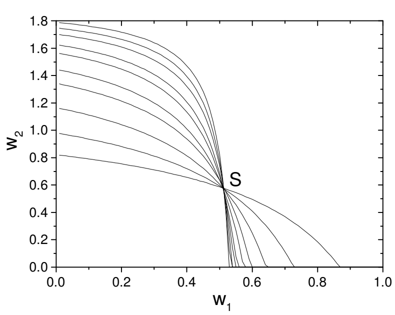

Fig.4. The hyperbola family of w_1 vs w_2 (d)

will increase exponentially. Taking into account the fact that $|K_2| < |K_1|, w_1(k)$ will converge to a and $w_2(k)$ to b very quickly.

Although practical signals are always nonstationary, they are stationary in short time period. Thus the above results can be used to analyze the convergence behavior of nonstationary processes in short time period. During the relatively stationary period of nonstationary processes, the separation parameters will converge quickly. But during the period of strong fluctuation of signals, the convergence will be destroyed and the divergence may take place. These phenomena are consistent with what we have seen in experiments.

5. SIMULATIONS

Two examples are presented to demonstrate the validity of the proposed algorithm.

5.1. Speech enhancement

In this example, one of the signals is a real speech signal, the other one is a computer-produced Gaussian noise with zero mean and 0.09 variance. The length of the data is 30788.

The mixing matrix is $M = \begin{bmatrix} 1.0 & 0.4 \\ 0.6 & 1.0 \end{bmatrix}$, forgetting factor is set to be ë0.999 for the estimation of the crosscorrelations.

The signal-to-noise ratios (SNR) of the two mixed signals are -4.39dB and -16.79dB respectively before separation; after separation, one of the separated signals is the speech, its signal-to-noise ratio is 30.42dB, the other separated signal is the background noise. The increment in SNR is about 34 dB. The separation result is given in Fig. 5. The result shows that the algorithm is very successful.

5.2. Speech separation

The two speech signals are given in Fig. 6. They are mixed up with a mixing matrix $M = \begin{bmatrix} 1.0 & 0.5 \\ 0.8 & 1.0 \end{bmatrix}$, We choose forgetting factors ë0.999. Before separation, the signal-to-interference ratios (SIR) are SIR_{x_1} = 5.64dB, SIR_{x_2} = 2.32dB. After separation, SIR_{y_1} = 29.44dB, SIR_{y_2} = 26.42dB. The increment in SNR is about 24dB. See the Fig. 6.

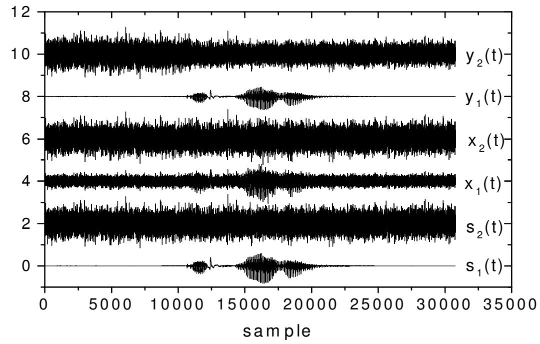

Fig.5. Speech enhancement. $s_1(t)$: Speech; $s_2(t)$: Background noise; $x_1(t), x_2(t)$: Two noise contaminated speeches; $y_1(t)$: Separated speech; $y_2(t)$: Separated noise.

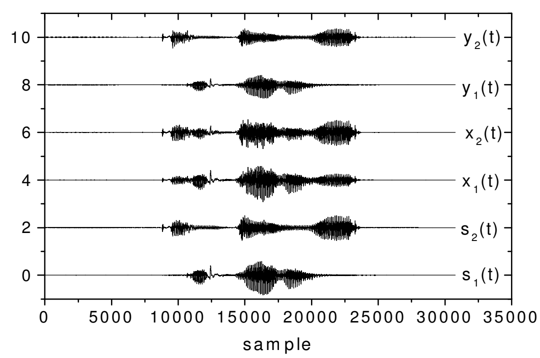

Fig.6. Speeches separation. $s_1(t), s_2(t)$: Source speeches; $x_1(t), x_2(t)$: Mixed speeches; $y_1(t)$, $y_2(t)$: Separated speeches.

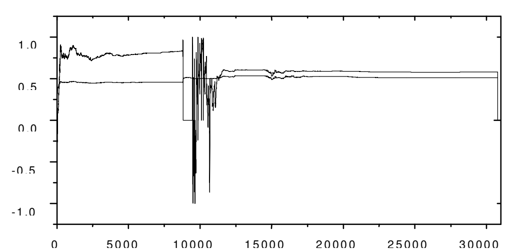

Fig.7. The learning curves of separation parameters w_1 and w_2.

The Fig. 7 is the learning curves of the separation parameters w_1 and w_2. At the beginning of speech signals, we really see the divergences of w_1 and w_2, but they converge to the correct solution quickly. This is consistent with the convergence analysis results in section 4.

6. CONCLUSIONS

In this paper, we have proposed an algorithm used for the separation of instantaneous mixtures of two sources based on decorrelation. It is almost the simplest one in BSS so far. The fast convergence property will make the proposed algorithm very useful in the separation of mixtures with time-variant parameters. Time-delayed decorrelation makes it possible to construct a noise-robust BSS algorithm because white noises can not affect the estimation of time-delayed crosscorrelations. The importance of nonstationary properties of sources has been emphasized in this paper. In addition, the algorithm is independent on the probability distribution of sources, both super-Gaussian and sub-Gaussian sources can be separated without any trouble. Even two independent white Gaussian noises can be separated with the aid of nonstationarities.

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