GENERALIZED DEFLATION ALGORITHMS FOR THE BLIND SOURCE-FACTOR SEPARATION OF MIMO-FIR CHANNELS

Mitsuru Kawamoto[1,2] *and Yujiro Inouye*[1]

1. Dept. of Electronic and Control Systems Engineering, Shimane University,
1060 Nishikawatsu, Matsue, Shimane 690-8504, Japan
2. Bio-Mimetic Control Research Center, RIKEN
2271-130 Anagahora, Moriyama-ku, Nagoay 463-0003, Japan

ABSTRACT

The present paper proposes a class of iterative deflation algorithms to solve the blind source-factor separation for the outputs of multiple-input multiple-output finite impulse response (MIMO-FIR) channels. Using one of the proposed deflation algorithms, filtered versions of the source signals, each of which is the contribution of each source signal to the outputs of MIMO-FIR channels, are extracted one by one from the mixtures of source signals. The proposed deflation algorithms can be applied to various sources, that is, i.i.d. signals, second-order white but higher-order colored signals, and second-order correlated (non-white) signals, which are referred to as *generalized deflation algorithms* (GDA's). The conventional deflation algorithms were proposed for each source signal mentioned above (e.g., [1, 5, 6, 8, 10]), that is, a class of deflation algorithms which can deal with each of the source signals mentioned above has not been proposed until now. Some simulation results are presented to show the validity of the proposed algorithms.

1. INTRODUCTION

This paper deals with the blind source-factor separation for the outputs of MIMO-FIR systems driven by colored source inputs, in which the contributions of source signals to the outputs of MIMO-FIR channels (contribution signals) are extracted from the mixtures of the source signals. To solve the blind source-factor separation problem, a class of iterative algorithms is proposed. The proposed algorithms are deflation algorithms which are obtained by modifying the super-exponential deflation algorithm proposed by Inouye and Tanebe [1] to the case of MIMO-FIR systems driven by colored source signals. Almost all conventional deflation algorithms have been applied to blind deconvolution problems under the assumption that source signals are i.i.d. signals (e.g., [1, 6, 10]). Recently, two algorithms were developed for extracting colored source signals from the mixtures of the source signals [5, 8]. They, however, can be applied only to the case when the source signals are colored signals but white in the sense of second-order statistics. This paper shows that the proposed deflation algorithms can be applied to the case when source signals are *temporally second-order correlated* (non-white) but spatially uncorrelated. Namely, even if the source signals are i.i.d. signals, second-order white but higher-order colored signals, or second-order correlated (non-white) signals, our proposed algorithms can achieve the blind source-factor separation. Such a class of deflation algorithms has not been proposed until now. The proposed deflation algorithms are referred to as *generalized deflation algorithms* (GDA's). Simulation examples are presented to illustrate the performance of the proposed algorithms.

2. FORMULATION OF A BLIND SOURCE-FACTOR SEPARATION PROBLEM

Throughout the present paper, let us consider the following MIMO-FIR system (channel):

$$\boldsymbol{x}(t) = \sum_{k=0}^{K} \boldsymbol{H}^{(k)} \boldsymbol{s}(t-k), \tag{1}$$

where $\boldsymbol{x}(t)$ represents an m-column output vector called the *observed signal*, $\boldsymbol{s}(t)$ represents an n-column input vector called the *source signal*, $\boldsymbol{H}^{(k)} = [h_{ij}^{(k)}]$ is an $m \times n$ matrix representing the impulse response of the channel, and the number K denotes its order. Eq. (1) can be rewritten as

$$\boldsymbol{x}(t) = \boldsymbol{H}(z)\boldsymbol{s}(t), \tag{2}$$

where $\boldsymbol{H}(z)$ is called the *transfer function*, which is defined by the z-transform of the impulse response $\{\boldsymbol{H}^{(k)}\}$, that is, $\boldsymbol{H}(z) = \sum_{k=0}^{K} \boldsymbol{H}^{(k)} z^k$. It is assumed for the sake of simplicity in the present paper that all the signals and the channel impulse responses are real-valued.

The blind source-factor separation problem is formulated as follows: The problem is to retrieve the contributions of each source to each output, that is, to find signals $c_{ij}(t)$'s $(i = 1, \cdots, m; j = 1, \cdots, n)$ defined as

$$c_{ij}(t) = h_{ij}(z)s_j(t), \tag{3}$$

where $h_{ij}(z)$ is the (i, j)-th element of $\boldsymbol{H}(z)$.

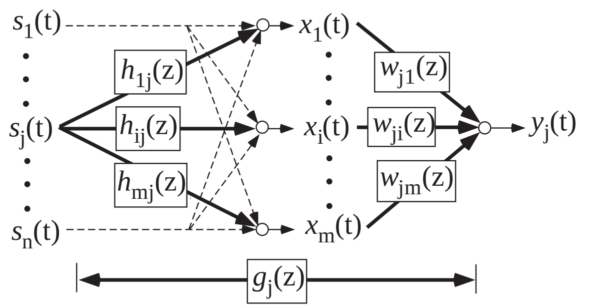

Figure 1: Cascade system of the unknown system and a filter.

Let us explain the scenario of solving this problem. First, we find a filtered version $y_j(t)$ of source $s_j(t)$ as

$$y_j(t) = g_j(z)s_j(t). \tag{4}$$

The filter $g_j(z)$ in (4) is given by $\sum_{i=1}^{m} w_{ji}(z)h_{ij}(z)$ (see Fig.1), where $w_{ji}(z) = \sum_{k=0}^{L} w_{ji}^{(k)} z^k$, and L is the order of $w_{ji}(z)$ and is a sufficiently large positive integer. Then, $g_j(z)$ cannot be directly handled, because $h_{ij}(z)$ is unknown. The filter $g_j(z)$ is found by adjusting the parameters $w_{ji}^{(k)}$'s of the filters $w_{ji}(z)$ $(i = 1, \cdots, m)$ as shown below, because $y_j(t)$ in (4) can be calculated by $y_j(t) = \sum_{i=1}^{m} w_{ji}(z)x_i(t)$. In this first step, we consider extracting the filtered source signal $y_j(t)$ from the observed signals $x_i(t)$ $(i = 1, \cdots, m)$, using a deflation approach [1]. Next, the contribution signals $c_{ij}(t)$ $(i = 1, \cdots, m)$ of the source signal $s_j(t)$ are calculated by using the filtered signal $y_j(t)$. Finally, we calculate $x_i(t) - c_{ij}(t)$ $(i = 1, \cdots, m)$. After all, by continuing these three steps until the last contribution is extracted, the blind source-factor separation problem is solved. On the details of the procedure, see Section 3.2.

The problem of finding $c_{ij}(t)$ can be considered as follows, when the filtered signals $y_j(t)$'s can be obtained. It follows from (3) and (4) that

$$c_{ij}(t) \quad = \quad a_{ij}(z)y_j(t), \tag{5}$$

where

$$a_{ij}(z) \quad = \quad h_{ij}(z)/g_j(z). \tag{6}$$

Therefore, it follows from (2), (4), and (6) that

$$\boldsymbol{x}(t) = \boldsymbol{A}(z)\boldsymbol{y}(t), \tag{7}$$

where $\boldsymbol{y}(t) = [y_1(t),\cdots,y_n(t)]^T$ and $\boldsymbol{A}(z) = [a_{ij}(z)]$. Thus, if the filtered versions $y_j(t)$ $(j = 1, \cdots, n)$ can be found, then the original problem of finding the contributions $c_{ij}(t)'s$ can be transformed into the problem of finding the transfer function $\boldsymbol{A}(z)$. Suppose the transfer function $\boldsymbol{A}(z)$ has no pole on the unit circle $|z|$=1. Then the problem of finding $\boldsymbol{A}(z)$ becomes a conventional (non-blind and non-causal) system-identification problem, because we know both of two signals $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$. Then, we can use the Wiener-Hopf technique (on the second-order correlation technique) [4] to find the transfer function $\boldsymbol{A}(z)$.

In the present paper, we choose that each $g_j(z)$ is an *Linear Phase* (*LP*) filter of the first type in which the impulse response sequence of the *LP* filter is symmetric for the center of the sequence [7] or a *Cascaded Integrator-Comb (CIC)* filter, that is,.

$$g_j(z) = \mathrm{g}_j(1 + z + \cdots + z^M), \;\; j = 1, \cdots, n, \tag{8}$$

where g_j is a real constant, and M is an appropriate order of $g_j(z)$. The *LP* property or the *CIC* property of each $g_j(z)$ is fulfilled by the procedure for constructing $g_j(z)$ mentioned below. We should note here that the inverses of an *LP* filter and a *CIC* filter are, respectively, unstable and not stable but critically stable. Because an *LP* filter has always zeros on the inside and the outside of the unit circle $|z| = 1$ [7] and a *CIC* filter has always zeros on the unit circle $|z| = 1$. This means that $\boldsymbol{A}(z)$ given by (7) is unstable or critically stable. However, we are interested in estimating $\boldsymbol{A}(z)$ in the present paper. Therefore, $\boldsymbol{A}(z)$ is approximated by a non-causal (double side) FIR transfer function of a sufficiently large order (see Step 4 in Section 3.2).

3. THE PROPOSED ALGORITHMS

3.1. How to construct the filters $g_j(z)$'s

The filters $g_j(z)$ $(j = 1, \cdots, n)$ are constructed by the two-step procedure mentioned below. Before stating the procedure, we introduce the following notation. Let us assume that the filters $g_j(z)$ $(j = 1, \cdots, n)$ are FIR filters as follows:

$$g_j(z) = \sum_{k=0}^{M} g_j^{(k)} z^k, \qquad j = 1, 2, \cdots, n, \tag{9}$$

where M is determined as $M = L + K$. For each j, we introduce the following $(M + 1)$-column vectors $\tilde{\boldsymbol{g}}_j$ and $\hat{\boldsymbol{g}}_j$ as, respectively,

$$\tilde{\boldsymbol{g}}_j := [g_j^{(0)}, \cdots, g_j^{(M)}]^T, \tag{10}$$

and

$$\hat{\boldsymbol{g}}_j := \left(\sum_{k=0}^{M} g_j^{(k)} \right)^p \mathbf{1}_{M+1}, \tag{11}$$

where $\mathbf{1}_{M+1}$ is an $(M + 1)$-column vector with all the elements being equal to 1 and p is a positive integer, the value of which is chosen by taking into account a class of source signals. (See Remark 2 for the details.) Note that $g_j^{(k)}$ in (11) is the $(k+1)$-st element of the vector $\tilde{\boldsymbol{g}}_j$ and that all the elements of the vector $\hat{\boldsymbol{g}}_j$ are equal to the p-th power of the sum of all the elements of the vector $\tilde{\boldsymbol{g}}_j$.

The vector $\tilde{\boldsymbol{g}}_j$ is iteratively determined by the following two-step procedure:

$$\tilde{\boldsymbol{g}}_j^{[1]} = \boldsymbol{\Sigma}_j^{-1}\kappa_j\hat{\boldsymbol{g}}_j, \qquad j=1,2,\cdots,n, \quad (12)$$

$$\tilde{\boldsymbol{g}}_j^{[2]} = \frac{\tilde{\boldsymbol{g}}_j^{[1]}}{\sqrt{\left|\sum_{j_1=1}^{n}\tilde{\boldsymbol{g}}_{j_1}^{[1]T}\boldsymbol{\Sigma}_{j_1}\tilde{\boldsymbol{g}}_{j_1}^{[1]}\right|}}, \quad j=1,2,\cdots,n, (13)$$

where $(\cdot)^{[1]}$ and $(\cdot)^{[2]}$, respectively, stand for the result of the first step and the result of the second step per iteration, $\boldsymbol{\Sigma}_j := \mathrm{E}[\boldsymbol{s}_j\boldsymbol{s}_j^T]$ is the correlation matrix of $\boldsymbol{s}_j = [s_j(0),\cdots,s_j(M)]^T$, and κ_j is the sum of all the $(p{+}1)$st-order auto-cumulants of the j-th source signal:

$$\kappa_j = \underbrace{\sum_{\tau_1,\cdots,\tau_{p-1},\tau_p\in Z}}_{p} \mathrm{Cum}\Big\{s_j(t), s_j(t-\tau_1),\cdots, s_j(t-\tau_{p-1}), s_j(t-\tau_p)\Big\} \neq 0 \quad (<\infty), \qquad (14)$$

where Z denotes the set of all integers and

$\mathrm{Cum}\{s_j(t), s_j(t-\tau_1),\cdots,s_j(t-\tau_{p-1}),s_j(t-\tau_p)\}$

is the $(p{+}1)$st-order auto-cumulant function of $s_j(t)$. Moreover, the $\hat{\boldsymbol{g}}_j$'s in (12) are defined by (11) and (10) for the $\tilde{\boldsymbol{g}}_j$'s.

The two-step procedure (12) and (13) becomes one cycle of iterations in the GDA's presented in the paper. To implement the two-step procedure, we need the following assumptions:

(A1) The FIR transfer function $\boldsymbol{H}(z)$ is *irreducible* [3], that is, $rank\ \boldsymbol{H}(z) = n$ for any $z \in C$ (this implies that $\boldsymbol{H}(z)$ has at least as many outputs as inputs ($n \leq m$) and is causal).

(A2) The input sequence $\{\boldsymbol{s}(t)\}$ is a zero-mean stationary vector process whose component processes $\{s_i(t)\}$ $(i=1,\cdots,n)$ have all positive-definite $\boldsymbol{\Sigma}_i$'s, non-zero κ_i's, and non-zero ρ_i's, where ρ_i denotes the sum of all the elements of the matrix $\boldsymbol{\Sigma}_i^{-1}$.

Let $\tilde{\boldsymbol{g}}_j(l)$ denotes the $(M+1)$-column vector obtained in the l-th cycle of the iterations of two steps (12) and (13), and $g_j^{(k)}(l)$ denotes the $(k+1)$st element of $\tilde{\boldsymbol{g}}_j(l)$ determined by the relation (10). The important thing of the two-step procedure is that the n vectors $\tilde{\boldsymbol{g}}_j(l)$ $(j=1,\cdots,n)$ converge to the zero vector except for only one of the vectors as the iteration number l approaches infinity (that is, $l\to\infty$). This will be shown in the following theorem.

Theorem 1: Let $\tilde{\boldsymbol{g}}_j(0) = [g_j^{(0)}(0),\cdots,g_j^{(M)}(0)]^T$ be an initial vector for iterations of two steps (12) and (13) for each $j=1,\cdots,n$. Let α_j be a non-negative scalar defined as

$$\alpha_j = \begin{cases} |\rho_j\kappa_j|^{\frac{1}{p-1}}\left|\sum_{k=0}^{M} g_j^{(k)}(0)\right| & \text{if } p\geq 2,\\ |\rho_j\kappa_j| & \text{if } p=1,\end{cases} \qquad (15)$$

for each $j=1,\cdots,n$. Let α_{j_0} be the largest of the scalars α_j's. Suppose it is only one, that is, $\alpha_{j_0} > \alpha_j$ for all $j\neq j_0$. Then, as $l\to\infty$, it follows

$$\lim_{l\to\infty}\tilde{\boldsymbol{g}}_j(l) = \begin{cases}\boldsymbol{0} & \text{for } j\neq j_0,\\ \bar{\boldsymbol{g}}_{j_0} & \text{for } j=j_0,\end{cases} \qquad (16)$$

where $\bar{\boldsymbol{g}}_{j_0} := [\bar{g}_{j_0}^{(0)},\cdots,\bar{g}_{j_0}^{(M)}]^T \neq \boldsymbol{0}$.

Proof of Theorem 1: From (12), choosing j_0 so that $\sum_{k=0}^{M} g_{j_0}^{(k)}(l)\neq 0$, we obtain,

$$\frac{|\hat{g}_j^{[1]}(l)|}{|\hat{g}_{j_0}^{[1]}(l)|} = \frac{|\rho_j\kappa_j|}{|\rho_{j_0}\kappa_{j_0}|}\left(\frac{|\hat{g}_j^{[1]}(l-1)|}{|\hat{g}_{j_0}^{[1]}(l-1)|}\right)^p, \qquad (17)$$

where $\hat{g}_j^{[1]}(l) := \sum_{k=0}^{M} g_j^{(k)[1]}(l)$ which is the element of $\hat{\boldsymbol{g}}_j$ and the integer l denotes the iteration time.

Let us consider (17) in case of $p\geq 2$. Note that $|\hat{g}_j^{[1]}(l)|/|\hat{g}_{j_0}^{[1]}(l)|$ is not modified by the normalization of the second step. Therefore, it is possible to solve $|\hat{g}_j^{[2]}(l)|/|\hat{g}_{j_0}^{[2]}(l)|$ from the recursive formula (17), which yields

$$\frac{|\hat{g}_j^{[2]}(l)|}{|\hat{g}_{j_0}^{[2]}(l)|} = \frac{|\rho_{j_0}\kappa_{j_0}|^{\frac{1}{p-1}}}{|\rho_j\kappa_j|^{\frac{1}{p-1}}} \cdot \left(\frac{|\rho_j\kappa_j|^{\frac{1}{p-1}}}{|\rho_{j_0}\kappa_{j_0}|^{\frac{1}{p-1}}}\frac{|\hat{g}_j^{[2]}(0)|}{|\hat{g}_{j_0}^{[2]}(0)|}\right)^{p^l} \quad (p\geq 2), \quad (18)$$

for any non-negative integer l. For j_0, that is,

$$j_0 = \arg\max_j |\rho_j\kappa_j|^{\frac{1}{p-1}}|\hat{g}_j^{[2]}(0)|,$$

one can see that all the other values $|\hat{g}_j^{[2]}(l)|$, $j\neq j_0$, quickly become small compared to $|\hat{g}_{j_0}^{[2]}(l)|$. Taking into account the normalization of the second step, this means that $|\hat{g}_{j_0}^{[2]}(l)|\neq 0$ and $|\hat{g}_j^{[2]}(l)|\to 0$ for all $j\neq j_0$. This implies that the infinite iteration of two steps (12) and (13) gives (16). Moreover, the equation (18) along with the normalization of the second step means that the sequence $\{\tilde{\boldsymbol{g}}_j(l)\}$ converges to the desired vector at a super-exponential rate when $p\geq 2$ for all $j=1,\cdots,n$.

Let us consider (17) in case of $p=1$. Then we can obtain the following equation.

$$\frac{|\hat{g}_j^{[2]}(l)|}{|\hat{g}_{j_0}^{[2]}(l)|} = \left(\frac{|\rho_j\kappa_j|}{|\rho_{j_0}\kappa_{j_0}|}\right)^l\frac{|\hat{g}_j^{[2]}(0)|}{|\hat{g}_{j_0}^{[2]}(0)|} \qquad (p=1) \quad (19)$$

Therefore, it can be seen that the convergence of the two-step procedure ($p = 1$) is decided by the ratio of $|\rho_j \kappa_j|/|\rho_{j_0} \kappa_{j_0}|$. Let $|\rho_{j_0} \kappa_{j_0}|$ be the largest of the scalars $|\rho_j \kappa_j|$ ($j = 1, \cdots, n$), that is,

$$j_0 = \arg\max_j |\rho_j \kappa_j|.$$

Then, taking into account the normalization of the second step, the equation (19) means that $|\hat{g}_{j_0}^{[2]}(l)| \neq 0$ and $|\hat{g}_j^{[2]}(l)| \to 0$ for all $j \neq j_0$. Moreover, the equation (19) along with the normalization of the second step means that the sequence $\{\tilde{\boldsymbol{g}}_j(l)\}$ converges to the desired vector at an exponential rate for all $j = 1, \cdots, n$. □

Let $\bar{g}_{j_0}(z)$ be the transfer function determined as

$$\bar{g}_{j_0}(z) = \sum_{k=0}^{M} \bar{g}_{j_0}^{(k)} z^k. \tag{20}$$

Then we have two properties of the filter $\bar{g}_{j_0}(z)$ as follows.

Theorem 2: Let $\bar{g}_{j_0}(z)$ be the transfer function determined by (20). Then $\bar{g}_{j_0}(z)$ possesses the following properties:
($P1$) It is the transfer function of an LP filter of the first type [7], if the source signal $s_{j_0}(t)$ is temporally second-order correlated.
($P2$) If the source signal $s_{j_0}(t)$ is temporally second-order white, then it is the transfer function of a *Cascaded Integrator Comb* (CIC) filter, that is,

$$\bar{g}_{j_0}(z) = \bar{\mathrm{g}}_{j_0}(1 + z + \cdots + z^M), \tag{21}$$

where $\bar{\mathrm{g}}_{j_0}$ is a scalar non-zero constant.

Proof of Theorem 2: The property ($P1$) is verified as follows: $\boldsymbol{\Sigma}_j^{-1}$ in (12) is a symmetric matrix, because $\boldsymbol{\Sigma}_j$ is a symmetric matrix. Therefore, since all elements of the vector $\hat{\boldsymbol{g}}_j$ in (12) are the same value and $\boldsymbol{\Sigma}_j^{-1}$ is a symmetric matrix, then the impulse responses $\bar{g}_{j_0}^{(k)}$'s satisfy $\bar{g}_{j_0}^{(k)} = \bar{g}_{j_0}^{(M-k)}$ ($k = 0, \cdots, M$). This implies that $\bar{g}_{j_0}(z)$ is an LP filter whose impulse response sequence is a symmetry for the center of the sequence.

The property ($P2$) is shown as follows: If the source signal $s_{j_0}(t)$ is temporally second-order white, then the covariance matrix $\boldsymbol{\Sigma}_{j_0}$ of $s_{j_0}(t)$ becomes $\boldsymbol{\Sigma}_{j_0} = \boldsymbol{D}$, where $\boldsymbol{D}$ is a diagonal matrix which has all the same non-zero diagonal elements. Then, (12) can be rewritten as follows;

$$\tilde{\boldsymbol{g}}_{j_0}^{[1]} = \boldsymbol{D}^{-1} \kappa_{j_0} \hat{\boldsymbol{g}}_{j_0}, \tag{22}$$

where κ_{j_0} denotes (14) of $s_{j_0}(t)$. Since $\hat{\boldsymbol{g}}_{j_0}$ has all the same elements, κ_{j_0} is a scalar constant, and $\boldsymbol{D}$ is a diagonal matrix whose diagonal elements are the same value, then $\bar{g}_{j_0}(z)$ becomes $\bar{g}_{j_0}(z) = \bar{\mathrm{g}}_{j_0}(1 + z + \cdots + z^M)$. Therefore, $\bar{g}_{j_0}(z)$ is the transfer function of a CIC filter. □

From Theorem 1, it can be proved that the two-step procedure (12) and (13) can be used to acquire a filtered version $y_j(t)$ in (4) from the observed signals $x_i(t)$ ($i = 1, \cdots, m$). In practice, however, one cannot directly handle $g_j(z)$, because $h_{ij}(z)$'s are unknown. Therefore, we must consider finding $g_j(z)$ using a filter $y_j(t) = \sum_{i=1}^m w_{ji}(z) x_i(t)$ (see Figure 1), where $w_{ji}(z) := \sum_{k=0}^L w_{ji}^{(k)} z^k$. This implies that we must transform the two steps (12) and (13) into two steps with respect to $w_{ji}^{(k)}$'s, that is,

$$\tilde{\boldsymbol{w}}_j^{[1]} = \tilde{\boldsymbol{R}}^{\dagger} \tilde{\boldsymbol{D}}_j, \qquad j = 1, 2, \cdots, n, \tag{23}$$

$$\tilde{\boldsymbol{w}}_j^{[2]} = \frac{\tilde{\boldsymbol{w}}_j^{[1]}}{\sqrt{|\tilde{\boldsymbol{w}}_j^{[1]T} \tilde{\boldsymbol{R}} \tilde{\boldsymbol{w}}_j^{[1]}|}}, \qquad j = 1, 2, \cdots, n, \tag{24}$$

where $\tilde{\boldsymbol{w}}_j$ is an $(L+1)m$-column vector defined by

$$\tilde{\boldsymbol{w}}_j := [\tilde{\boldsymbol{w}}_{j1}^T, \tilde{\boldsymbol{w}}_{j2}^T, \cdots, \tilde{\boldsymbol{w}}_{jm}^T]^T, \tag{25}$$

$$\tilde{\boldsymbol{w}}_{ji} := [w_{ji}^{(0)}, w_{ji}^{(1)}, \cdots, w_{ji}^{(L)}]^T, \tag{26}$$

where † denotes the pseudo-inverse operation of a matrix, $\tilde{\boldsymbol{R}}$ is the $m \times m$ block matrix defined by

$$\tilde{\boldsymbol{R}} := \begin{bmatrix} \tilde{\boldsymbol{R}}_{11} & \tilde{\boldsymbol{R}}_{12} & \cdots & \tilde{\boldsymbol{R}}_{1m} \\ \tilde{\boldsymbol{R}}_{21} & \tilde{\boldsymbol{R}}_{22} & \cdots & \tilde{\boldsymbol{R}}_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\boldsymbol{R}}_{m1} & \tilde{\boldsymbol{R}}_{m2} & \cdots & \tilde{\boldsymbol{R}}_{mm} \end{bmatrix} \tag{27}$$

whose (i, j)-th block element $\tilde{\boldsymbol{R}}_{ij}$ is the $(L+1) \times (L+1)$ matrix with the (i_1, j_1)-th element $[\tilde{\boldsymbol{R}}_{ij}]_{i_1 j_1}$ defined by

$$[\tilde{\boldsymbol{R}}_{ij}]_{i_1 j_1} = \mathrm{Cum}\{x_j(t - j_1), x_i(t - i_1)\}, \tag{28}$$

and $\tilde{\boldsymbol{D}}_j$ is the m-block vector defined by

$$\tilde{\boldsymbol{D}}_j := [\tilde{\boldsymbol{d}}_{j1}^T, \tilde{\boldsymbol{d}}_{j2}^T, \cdots, \tilde{\boldsymbol{d}}_{jm}^T]^T, \tag{29}$$

where $\tilde{\boldsymbol{d}}_{ji}$-th is the $(L+1)$-column vector with the j_1-th element $[\tilde{\boldsymbol{d}}_{ji}]_{j_1}$ given by

$$[\tilde{\boldsymbol{d}}_{ji}]_{j_1} = \sum_{\underbrace{\tau_1, \cdots, \tau_{p-1}, \tau_p}_{p} \in Z} \mathrm{Cum}\Big\{ y_j(t), y_j(t - \tau_1), \cdots, y_j(t - \tau_{p-1}), x_i(t - j_1 - \tau_p) \Big\}. \tag{30}$$

Remark 1: The first step (23) and the second step (24) correspond to (12) and (13), respectively. In particular, (23) can be derived by solving a weighted least squares problem in the $\tilde{\boldsymbol{w}}_j$-domain for each $j = 1, 2, \cdots, n$. See [1] for the details.

Remark 2: It can be seen from (12) and (14) that the value of p in (11) relates to the order of the auto-cumulant of $s_j(t)$ in κ_j. Hence, the value of p in (11) can be changed according to how the statistical properties of source signals are taken into account. This implies that if one chooses $p = 1$, the second-order statistics of source signals are used to solve the blind source-factor separation problem. Therefore, if the source signals are temporally second-order correlated, the blind source-factor separation problem can be solved by setting to $p = 1$. On the other hand, if the source signals are temporally second-order white (e.q., i.i.d. signal), one must consider the problem by using (12) in case of $p \geq 2$.

3.2. The generalized deflation algorithms (GDA's)

A deflation approach is used to implement the blind source-factor separation. First we choose the value of p based on a class of source signals. Then the proposed deflation algorithm, that is, the GDA with given p for the blind source-factor separation can be summarized in the following steps:

Step 1. Set $j = 1$ (where j denotes the order of the filtered version of an input extracted).

Step 2. Let $\tilde{\boldsymbol{w}}_j(l)$ denotes the $(L+1)m$-column vector obtained in the l-th cycle of the iterations of two steps (23) and (24). Set $l = 0$. Choose random initial values $\tilde{\boldsymbol{w}}_j(0)$, and then calculate $\tilde{\boldsymbol{w}}_j(0)/\sqrt{|\tilde{\boldsymbol{w}}_j(0)^T\tilde{\boldsymbol{R}}\tilde{\boldsymbol{w}}_j(0)|}$.

Step 3. Calculate $\tilde{\boldsymbol{w}}_j(l)$ using (23) and (24). The expectations can be estimated using some data samples of $y_j(t)$ and $x_i(t)$.

Step 4. At this stage, we assume that a filtered source signal $y_j(t) = \bar{g}_{j_0}(z)s_{j_0}(t)$ can be recovered. To remove the contribution of $s_{j_0}(t)$ to the observed signals $x_i(t)$ $(i = 1, \cdots, m)$, the criterion $E[\{x_i(t) - d_{ij}(z)y_j(t)\}^2]$, where $d_{ij}(z)y_j(t) = \sum_{k=-L'}^{L'} d_{ij}{}^{(k)}y_j(t-k)$ and the number L' is its order which is a sufficiently large, is minimized by adapting the filter $d_{ij}(z)$ [8]. Then, $x_i{}^{(j)} = x_i(t) - d_{ij}(z)y_j(t)$ $(i = 1, \cdots, m)$ are the outputs of a linear unknown multichannel system with m outputs and $n-1$ inputs. And the signals $d_{ij}(z)y_j(t)$'s denote the desired signals $c_{ij}(t)$'s in (3).

Step 5. If the superscript (j) of $x_i{}^{(j)}(t)$ is less than n, then set $j = j+1$ and $x_i(t) = x_i{}^{(j)}$ $(i = 1, \cdots, m)$ and go to Step 2. If it is equal to $n-1$, then stop here.

Remark 3: In Step 4, if the source signal $s_{j_0}(t)$ is temporally second-order white, then $s_{j_0}(t)$ can be obtained by implementing the whitening of the output $y_j(t) = \bar{g}_{j_0}(z)s_{j_0}(t)$, using an AR filter.

4. SIMULATION RESULTS

To demonstrate the validity of the GDA's, many computer simulations were conducted. Some results are shown in this section. We considered the following two-input and three-output FIR system.

$$\boldsymbol{H}(z) = \begin{bmatrix} 1.0 + 0.6z + 0.3z^2 & 0.6 + 0.5z - 0.2z^2 \\ 0.5 - 0.1z + 0.2z^2 & 0.3 + 0.4z + 0.5z^2 \\ 0.7 + 0.1z + 0.4z^2 & 0.1 + 0.2z + 0.1z^2 \end{bmatrix}. \quad (31)$$

It should be noted that $\boldsymbol{H}(z)$ satisfies $(A1)$. The observed signals $\boldsymbol{x}(t) = [x_1(t), x_2(t), x_3(t)]^T$ $(t = 0, 1, 2, \cdots)$ were calculated by (2). We used (30) with a chosen p.

[**Example 1**] In this example, the source signals $\{s_i(t)\}$ $(i = 1, 2)$ were generated by using the following system:

$$\begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{1-0.5z} & 0 \\ 0 & \frac{0.7+z}{1+0.7z} \end{bmatrix} \begin{bmatrix} \nu_1(t) \\ \nu_2(t) \end{bmatrix}. \quad (32)$$

where $\{\nu_1(t)\}$ and $\{\nu_2(t)\}$ were non-Gaussian i.i.d. signals with zero mean and unit variance, but were independent with each other. The source signal $\{s_1(t)\}$ is a non-white signal (temporally second-order correlated). Since the filter $(0.7 + z)/(1 + 0.7z)$ in (32) was a all-pass filter, the source signal $\{s_2(t)\}$ became temporally second-order white but temporally higher-order colored signals. We chose $p = 3$. The values of τ_1, τ_2, and τ_3 in (30) were set to belong to the intervals [0,10], [0,10], and [0,10], respectively.

We calculated the separation rate of the jth contribution on the ith output defined by

$$SEPR_{ij} = \frac{\sum_t |\bar{c}_{ij}(t) - c_{ij}(t)|^2}{\sum_t |c_{ij}(t)|^2}, \quad (33)$$

where $c_{ij}(t)$ denotes the desired signal in (3) and $\bar{c}_{ij}(t)$ denotes the estimated contribution using the proposed deflation algorithm. Then we obtained $SEPR_{11} = 0.0153$ and $SEPR_{12} = 0.0584$. In order to measure the "degree of improvement" of our algorithm, we note that before the source separation procedure, $SEPR_{bef_{11}} = 0.2610$ and $SEPR_{bef_{12}} = 3.8308$, where $SEPR_{bef_{ij}}$ is

$$SEPR_{bef_{ij}} = \frac{\sum_t |x_i(t) - c_{ij}(t)|^2}{\sum_t |c_{ij}(t)|^2}. \quad (34)$$

From this result, one can see that the GDA with $p = 3$ can be used to recover the desired signals $c_{ij}(t)'$s in (3).

[**Example 2**] In this example, each source signal $\{s_i(t)\}$ was a speech signal. We used (30) in case of $p = 1$. The values of τ_1 in (30) was set to [0,30].

Figure 2 shows the desired signals $c_{11}(t)$ and $c_{12}(t)$ in (3), and the estimated contributions $\bar{c}_{11}(t)$ and $\bar{c}_{12}(t)$ using the GDA ($p = 1$). From Figure 2, one can see that the GDA with $p = 1$ can be used to recover the desired signals $c_{ij}(t)'$s in (3). Then, calculating the separation rate (33) of the jth contribution on the ith

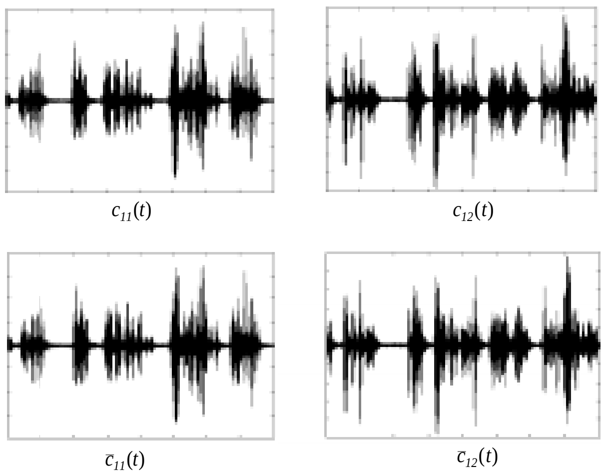

Figure 2: The desired signals $c_{11}(t)$ and $c_{12}(t)$ (upper), and the estimated contributions $\bar{c}_{11}(t)$ and $\bar{c}_{12}(t)$ (down)

output, we obtained $SEPR_{11} = 0.0015$ and $SEPR_{12} = 0.0039$. And calculating (34), we obtained $SEPR_{bef_{11}} = 0.3781$ and $SEPR_{bef_{12}} = 2.645$. See [11] for other simulation results.

5. CONCLUSIONS

The present paper has proposed a class of iterative deflation algorithms for the blind source-factor separation problem. The proposed deflation algorithms, that is, GDA's in case of $p \geq 2$ were super-exponential deflation algorithms (like [1]) mentioned in the proof of Theorem 1, and the GDA in case of $p = 1$ was an exponential deflation algorithm mentioned in the proof of Theorem 1. The GDA's were used to extract filtered version sources from the mixtures of sources.

It has been confirmed by many computer simulations that the GDA's can be used successfully to achieve the blind source-factor separation. Some results were shown in the present paper. In this simulation results, it can also be confirmed that the GDA's are globally convergent almost always (this result is similar to the one shown in [1]).

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