

ESTIMATING AR PARAMETER-SETS FOR LINEAR-RECURRENT SIGNALS IN CONVOLUTIVE MIXTURES

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ABSTRACT

This article investigates a theoretical basis for estimating autoregressive (*AR*) processes for linear-recurrent signals in convolutive mixtures. Whitening of such signals is sometimes a problem in multichannel blind equalization which is intended to extract the original signals even if the signals are of a convolutive mixture type. This whitening is due to inverse-filtering which deconvolves the *AR* processes that generate the linear-recurrent signals. To avoid this excessive deconvolution, it is effective to remove the contributions of such processes from the inverse-filters. Unfortunately, no method seems to have been able to extract the *AR* parameter-sets for respective signals included in convolutive mixtures.

1. INTRODUCTION

In multichannel blind equalization, or equivalently time-domain blind source separation, various methods have been proposed to extract the original signals in convolutive mixtures. One such method is based on blind inverse-filtering in an independent component analysis framework [1-3], where the signals are assumed to be mutually independent, zero-mean, and temporally *iid*. If signals are colored, however, difficulties arise due to whitening of such signals [4-6]. This is because the processes that generate such colored signals are deconvolved through inverse-filtering together with the signal transmission-channels from signal sources to sensors. An effective method to avoid this excessive deconvolution is to remove the contributions of such generative processes from the inverse-filters [5, 7].

To this end, if colored signals are linear-recurrent, their autoregressive (*AR*) processes [8] should be estimated. However, no method has successfully extracted the *AR* parameter-set for a signal that is of a convolutive mixture type. Conventional linear prediction (*LP*) methods may also whiten linear-recurrent signals as in the inverse-filtering case [9, 10, 22].

This article is intended to show a theoretical possibility of estimating respective *AR* processes of linear-recurrent

signals in convolutive mixtures of the signals. This possibility is derived from a similar framework to conventional multichannel *LP* methods, and is based on the following conditions:

- Each signal is generated through a finite *AR* process from its innovation (prediction error) [7, 11]. The innovations are mutually independent, zero-mean and temporally *iid*.
- Signal-transmission channels from signal sources to sensors are finite support, and a $p * q$ ($p < q$) polynomial matrix of such channel transfer functions has full (row-) rank, where p and q denote the number of signal sources and sensors, respectively.
- Both *AR* processes and signal-transmission channels are time invariant, or in other words, they are stationary over the period of estimating the processes and the channels.

The remainder of this article is organized as follows. In Section 2, a review of conventional multichannel *LP* method is given. *Interestingly, linear-recurrent signals are completely whitened so that they return to their innovations.* In Section 3, a theoretical method is proposed for estimating the *AR* parameter-sets for respective signals included in convolutive mixtures. An application of the proposed method to multichannel blind equalization is also remarked.

2. REVIEW OF MULTICHANNEL LP METHOD

Figure 1 shows a schematic diagram of a multichannel *LP* method that is intended to extract the innovation, $e_i(k)$, of a linear-recurrent signal $u_i(k)$ from convolutive mixtures of signals $u_1(k)$ and $u_2(k)$, where k denotes an integer index. These signals are assumed to be mutually independent, and respectively generated from finite *AR* processes from their innovations. Each signal is detected with sensors M_j after being convolved through polynomial transfer-functions $g_{ij}(z^{-1})$ from signal sources S_i to the sensors, where $i = 1, 2$ and $j = 1, 2, 3$ in this case. The one-step delayed output signals of the sensors are processed through a linear

predictor-set $\mathbf{w}(z^{-1})$ resulting in a good estimate of the output signal of the sensor M_j .

Suppose here that the signal $u_1(k)$ arrives at the sensor M_1 earlier than the other two sensors. Similarly, the signal $u_2(k)$ arrives at the sensor M_2 earlier than the others. Moreover, transfer functions $g_{ij}(z^{-1})$ are simplified as

$$h_{ij}(z^{-1}) = g_{ij}(z^{-1})/d_i \quad \text{for } i = 1, 2 \text{ and } j = 1, 2, 3, \quad (1)$$

where

d_i denotes the shortest signal propagation delay among those from the signal source S_i to the sensors M_j .

Then, the linear predictor-set $\mathbf{w}(z^{-1})$ may be obtained by minimizing the mean square value of an error $e(k)$ given as

$$e(k) = \mathbf{h}_1(z^{-1})^T \mathbf{u}(k) - \mathbf{w}(z^{-1})^T \mathbf{H}(z^{-1})^T \mathbf{u}(k-L), \quad (2a)$$

$$\mathbf{u}(k) = [u_1(k), u_2(k)]^T, \quad (2b)$$

$$\mathbf{w}(z^{-1}) = [w_1(z^{-1}), w_2(z^{-1}), w_3(z^{-1})]^T, \quad (2c)$$

$$\mathbf{h}_j(z^{-1}) = [h_{1j}(z^{-1}), h_{2j}(z^{-1})]^T \quad \text{for } j = 1, 2, 3, \text{ and} \quad (2d)$$

$$\mathbf{H}(z^{-1}) = [\mathbf{h}_1(z^{-1}), \mathbf{h}_2(z^{-1}), \mathbf{h}_3(z^{-1})]^T, \quad (2e)$$

where

$$u_i(k) = a_{i,0}u_i(k-L) + \dots + a_{i,m+L}u_i(k-(m+L+1)) + e_i(k) \quad (i = 1, 2),$$

$a_{i,p}$ is an AR coefficient ($p = 0, 1, \dots, m+L$),

m denotes the degree of polynomial $h_{ij}(z^{-1})$,

$\mathbf{H}(z^{-1})$ is a full row-rank polynomial matrix of $\mathbf{h}_j(z^{-1})$,

$e_i(k)$ is the innovation of $u_i(k)$, and

$w_j(z^{-1})$ is a linear predictor with a degree of L .

As for the signal $u_1(k)$ detected with the sensor M_1 ,

$$h_{11}(z^{-1})u_1(k) \hat{=} h_{11,0}u_1(k) + \mathbf{h}_{11:1-m}(z^{-1})u_1(k-1), \quad (3)$$

the linear predictor-set $\mathbf{w}(z^{-1})$ may predict the “direct component,” $h_{11,0}u_1(k)$, and replicate the “reverberant component,” $\mathbf{h}_{11:1-m}(z^{-1})u_1(k-1)$, from the one-step delayed output signals of the sensors. The linear predictor-set may also replicate the signal $u_2(k)$ detected with the sensor M_1 [12, 13]. Consequently, the error $e(k)$ is made equivalent to the innovation $e_1(k)$, except for its amplitude. The innovation $e_2(k)$ may also be extracted from the evaluation as

$$\min_w E\{|e(k)|^2\} = \min_w E\{|\mathbf{h}_1(z^{-1})^T \mathbf{u}(k) - \mathbf{w}(z^{-1})^T \mathbf{H}(z^{-1})^T \mathbf{u}(k-L)|^2\}, \quad (4)$$

where

$E\{x\}$ denotes the averaged (expected) value of a variable x .

Relations (2a-e) may be expressed in matrix form as

$$e(k) = \mathbf{h}_1^T \mathbf{u}_k - \mathbf{w}^T \mathbf{H}^T \mathbf{u}_{k-L}, \quad (5a)$$

or equivalently,

$$e(k) = \mathbf{u}_k^T \mathbf{h}_1 - \mathbf{u}_{k-L}^T \mathbf{H} \mathbf{w}, \quad (5b)$$

$$\mathbf{u}_k = [\mathbf{u}_{1:k}^T, \mathbf{u}_{2:k}^T]^T, \quad (5c)$$

$$\mathbf{w} = [\mathbf{w}_1^T, \mathbf{w}_2^T, \mathbf{w}_3^T]^T, \quad (5d)$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{11}, & \mathbf{H}_{12}, & \mathbf{H}_{13} \\ \mathbf{H}_{21}, & \mathbf{H}_{22}, & \mathbf{H}_{23} \end{bmatrix}, \quad (5e)$$

$$\mathbf{h}_1 = [[h_{11:0}, \dots, h_{11:m}, 0, \dots, 0], [0, h_{21:1}, \dots, h_{21:m}, 0, \dots, 0]]^T,$$

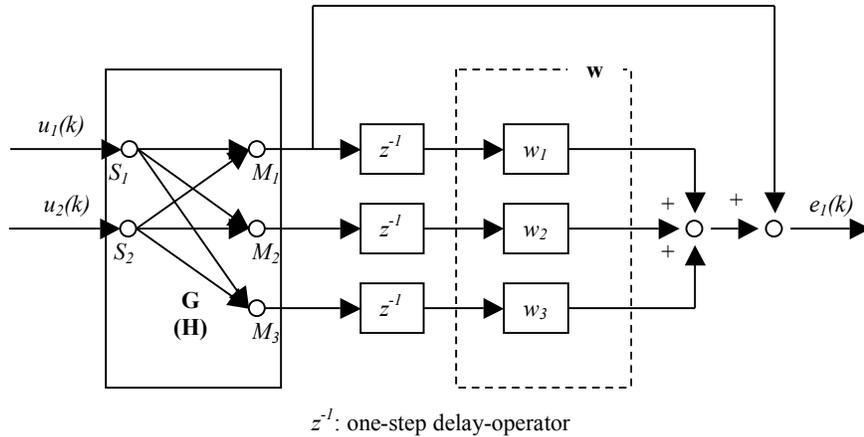


Fig. 1 Schematic diagram of a multichannel linear prediction system.

The linear predictors w_j ($j = 1, 2, 3$) are calculated so as to minimize the averaged square of the output signal. Consequently, the output signal may become equivalent to the innovation $e_1(k)$ of the signal $u_1(k)$. On the other hand, the signal $u_2(k)$ may precisely be eliminated.

$$\hat{=} [[h_{11:0}, \mathbf{h}_{11:1-m}^T], [0, \mathbf{h}_{21:1-m}^T]]^T, \quad (5f)$$

where

$$\mathbf{u}_{i:k} = [u_i(k), \dots, u_i(k - (m+L))]^T \quad \text{for } i = 1, 2,$$

$$\mathbf{u}_{i:k} = \begin{pmatrix} a_{i:0} & a_{i:1} & \dots & a_{i:m+L} \\ 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{pmatrix} \mathbf{u}_{i:k-1} + \begin{pmatrix} e_i(k) \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix},$$

$$\hat{=} \mathbf{C}_i \mathbf{u}_{i:k-1} + \mathbf{e}_{i:k-1},$$

\mathbf{C}_i is a companion matrix [7,14],

\mathbf{w}_j is a linear-predictor vector with a dimension of $L - 1$,

\mathbf{H} is a full row-rank matrix of \mathbf{H}_{ij} for $j = 1, 2, 3$,

\mathbf{H}_{ij} is an $(m+L+1) * L$ convolution matrix such as

$$\mathbf{H}_{ij} = \begin{pmatrix} h_{ij:0} & 0 & \dots & 0 \\ h_{ij:1} & h_{ij:0} & \dots & 0 \\ \vdots & h_{ij:1} & \dots & \vdots \\ h_{ij:m} & \vdots & \dots & h_{ij:0} \\ 0 & h_{ij:m} & \dots & h_{ij:1} \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & h_{ij:m} \end{pmatrix},$$

$h_{ij:p}$ is a coefficient of the polynomial $h_{ij}(z^{-1})$ for $p = 0, 1, \dots, m$,

and

\mathbf{h}_1 denotes the first column vector of the matrix \mathbf{H} , which reflects the assumption that signals $u_1(k)$ and $u_2(k)$ are supposed to arrive first at the sensors M_1 and M_2 , respectively.

Since the signals $u_1(k)$ and $u_2(k)$ are assumed mutually independent, the linear predictor-set \mathbf{w} may be calculated as

$$\begin{aligned} \mathbf{w} &= \{\mathbf{H}^T \mathbf{E} \{\mathbf{u}_{k-l} \mathbf{u}_{k-l}^T\} \mathbf{H} + s^2 \mathbf{I}\}^{-1} \mathbf{H}^T \mathbf{E} \{\mathbf{u}_{k-l} \mathbf{u}_k^T\} \mathbf{h}_1, \\ &= \{\mathbf{H}^T \mathbf{E} \{\mathbf{u}_{k-l} \mathbf{u}_{k-l}^T\} \mathbf{H} + s^2 \mathbf{I}\}^{-1} \mathbf{H}^T \mathbf{E} \{\mathbf{u}_{k-l} \mathbf{u}_{k-l}^T\} \mathbf{C}^T \mathbf{h}_1, \end{aligned} \quad (6)$$

where

s is an arbitrary small positive number,

\mathbf{C} denotes a block diagonal matrix of the companion matrices

\mathbf{C}_i for $i = 1, 2$, which satisfies the relation;

$$\mathbf{E} \{\mathbf{u}_{k-l} \mathbf{u}_k^T\} = \mathbf{C} \mathbf{E} \{\mathbf{u}_{k-l} \mathbf{u}_{k-l}^T\}.$$

If a positive number s is set to be sufficiently small, this relation may be simplified as [15, 22]

$$\begin{aligned} \underline{\mathbf{w}} &= \lim_{s \rightarrow 0} \mathbf{w}, \\ &= (\mathbf{U}_{k-l}^T \mathbf{H})^+ \mathbf{U}_{k-l}^T \mathbf{C}^T \mathbf{h}_1, \\ &= \mathbf{H}^T (\mathbf{H} \mathbf{H}^T)^{-1} (\mathbf{U}_{k-l} \mathbf{U}_{k-l}^T)^{-1} (\mathbf{U}_{k-l} \mathbf{U}_{k-l}^T) \mathbf{C}^T \mathbf{h}_1, \\ &= \mathbf{H}^T (\mathbf{H} \mathbf{H}^T)^{-1} \mathbf{C}^T \mathbf{h}_1, \end{aligned} \quad (7)$$

where

$$\mathbf{U}_{k-l} \mathbf{U}_{k-l}^T \hat{=} \mathbf{E} \{\mathbf{u}_{k-l} \mathbf{u}_{k-l}^T\},$$

\mathbf{U}_{k-l} is a matrix with a rank of $2(m+L+1)$, and

\mathbf{X}^+ denotes the Moore-Penrose pseudo inverse of a matrix \mathbf{X} .

The error $e(k)$ may consequently be expressed as

$$\begin{aligned} e(k) &= \mathbf{u}_k^T \mathbf{h}_1 - \mathbf{u}_{k-l}^T \mathbf{H} \underline{\mathbf{w}}, \\ &= (\mathbf{u}_k^T - \mathbf{u}_{k-l}^T \mathbf{C}^T) \mathbf{h}_1, \\ &= [[e_1(k), 0, \dots], [e_2(k), 0, \dots]] [[h_{11:0}, \mathbf{h}_{11:1-m}^T], [0, \mathbf{h}_{21:1-m}^T]]^T, \\ &= h_{11:0} e_1(k). \end{aligned} \quad (8a)$$

This relation demonstrates that the innovation $e_1(k)$ of the signal $u_1(k)$ may be estimated up to a scalar multiple. The "eliminations of the signal $u_2(k)$ and the "reverberant component" of the signal $u_1(k)$, $h_{11:1-m}(z^{-1})u_1(k-1)$ or equivalently $\mathbf{u}_{1:k-1}^T [h_{11:1-m}^T, 0]^T$, are clarified from another expression of relation (8a) as follows.

$$\begin{aligned} e(k) &= \mathbf{u}_k^T \mathbf{h}_1 - \mathbf{u}_{k-l}^T \mathbf{C}^T \mathbf{h}_1, \\ &= [\mathbf{u}_{1:k}^T, \mathbf{u}_{2:k}^T] [[h_{11:0}, \mathbf{h}_{11:1-m}^T], [0, \mathbf{h}_{21:1-m}^T]]^T \\ &\quad - [\mathbf{u}_{1:k-1}^T, \mathbf{u}_{2:k-1}^T] [h_{11:0} \mathbf{a}_1^T + [\mathbf{h}_{11:1-m}^T, 0], [\mathbf{h}_{21:1-m}^T, 0]]^T, \\ &= h_{11:0} \{u(k) - \mathbf{u}_{1:k-1}^T \mathbf{a}_1\} \\ &\quad - [\mathbf{u}_{1:k-1}^T, \mathbf{u}_{2:k-1}^T] \{[[h_{11:1-m}^T, 0], [\mathbf{h}_{21:1-m}^T, 0]]^T \\ &\quad - [[\mathbf{h}_{11:1-m}^T, 0], [\mathbf{h}_{21:1-m}^T, 0]]^T\}, \\ &= h_{11:0} e_1(k), \end{aligned} \quad (8b)$$

where

\mathbf{a}_i denotes a column vector of the AR coefficients $a_{i:p}$ for $i = 1, 2$, and $p = 0, 1, \dots, m+L$.

3. ESTIMATING AR PARAMETER-SETS

Relation (7) may be rewritten by replacing the column vector \mathbf{h}_1 with the matrix \mathbf{H} as

$$\underline{\mathbf{W}} = \mathbf{H}^T (\mathbf{H} \mathbf{H}^T)^{-1} \mathbf{C} \mathbf{H}. \quad (9a)$$

Similar to the linear predictor-set $\underline{\mathbf{w}}$, the matrix $\underline{\mathbf{W}}$ may be obtained by minimizing the mean average value of an error vector \mathbf{e}_k that is given by replacing the vector \mathbf{h}_1 with the matrix \mathbf{H} in relation (5b).

$$\mathbf{e}_k = \mathbf{u}_k^T \mathbf{H} - \mathbf{u}_{k-l}^T \mathbf{H} \underline{\mathbf{W}} \quad (9b)$$

Note here that the linear predictor-set $\underline{\mathbf{w}}$ given by relation (7) appears as the first column vector of the matrix $\underline{\mathbf{W}}$. Then, we may find the following relation among non-zero eigenvalues of the matrices $\underline{\mathbf{W}}$ and \mathbf{C} [16].

$$\begin{aligned} l_p \{\underline{\mathbf{W}}\} &= l_p \{\mathbf{H}^T (\mathbf{H} \mathbf{H}^T)^{-1} \mathbf{C} \mathbf{H}\}, \\ &= l_p \{\mathbf{H} \mathbf{H}^T (\mathbf{H} \mathbf{H}^T)^{-1} \mathbf{C}\}, \\ &= l_p \{\mathbf{C}\}, \end{aligned} \quad (10)$$

where

p is an integer index not larger than $2(m+L+1)$.

Since the rational matrix \mathbf{C} is a direct sum of the companion matrices \mathbf{C}_1 and \mathbf{C}_2 (see relations (5a-f) and (6)), the eigenvalues of the rational matrix contains all the eigenvalues of the companion matrices. In other words, the characteristic polynomial of the rational matrix is given as a product of those of the companion matrices, except for ambiguous sign. Since this ambiguousness may not cause any substantial difference in the following investigation, the ambiguousness will be ignored hereafter. Note here that such polynomial of each companion matrix \mathbf{C}_i denotes the prediction error filter (PEF) for generating the signal $u_i(k)$ as [17]

$$a_i(z^{-1}) = 1 - \{a_{i:0}z^{-1} + \dots + a_{i:m+L}z^{-(m+L+1)}\} \quad \text{for } i = 1, 2. \quad (11)$$

The characteristic polynomial of the matrix \mathbf{W} is, therefore, equivalent to the product of the PEFs for the signals $u_1(k)$ and $u_2(k)$. Next, we propose a possible derivation of the PEF for the signal $u_1(k)$.

If the signal $u_2(k)$ is white noise, the PEF $a_2(z^{-1})$ for this signal will be unity. The characteristic polynomial, $c_w(z^{-1})$, of the matrix \mathbf{W} may consequently be equivalent to the PEF for the signal $u_1(k)$ as

$$c_w(z^{-1}) = a_1(z^{-1}). \quad (12a)$$

If, on the other hand, the signal $u_2(k)$ is linear-recurrent, the characteristic polynomial of the matrix \mathbf{W} may be given as a product of the PEFs for the signals $u_1(k)$ and $u_2(k)$ such as

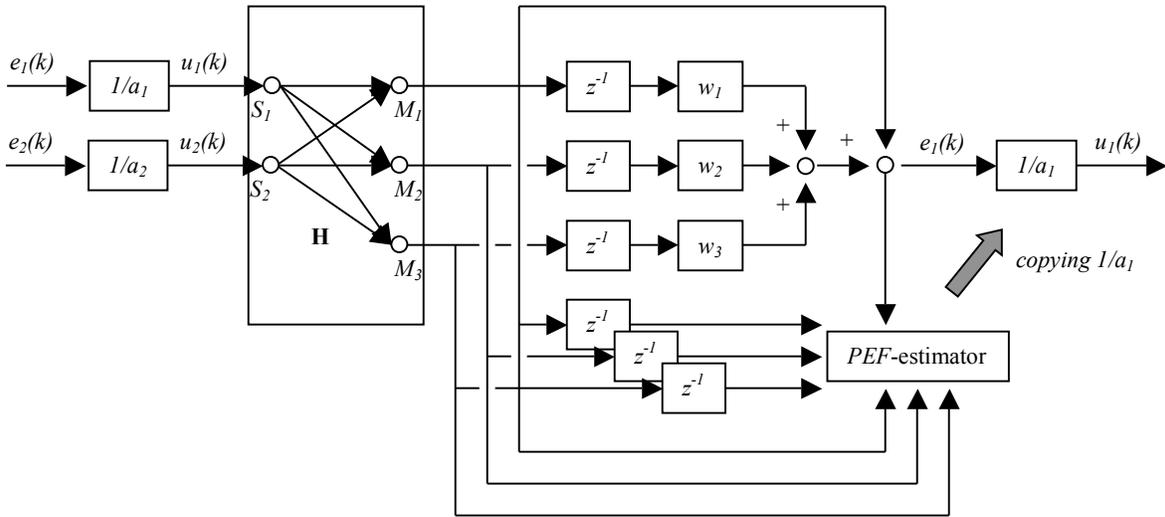
$$c_w(z^{-1}) = a_1(z^{-1}) a_2(z^{-1}). \quad (12b)$$

To simplify the investigation hereafter, the PEFs $a_1(z^{-1})$ and $a_2(z^{-1})$ are supposed to be coprime. Since these PEFs are minimum-phase, the inverse of the characteristic polynomial $c_w(z^{-1})$ may be expressed as

$$c_w(z^{-1}) = \{1/a_1(z^{-1})\} \{1/a_2(z^{-1})\}. \quad (12c)$$

The polynomial transfer function inclusive of the inverse PEF $1/a_1(z^{-1})$, $f_i(z^{-1})$, between the signal source S_i and the sensor M_i may be identifiable from the simple cross-correlation of the innovation $e_i(k)$ and the output signal of the sensor, $\mathbf{h}_1(z^{-1})^T \mathbf{u}(k) = h_{11}(z^{-1})u_1(k) + h_{21}(z^{-1})u_2(k)$, as follows.

$$\begin{aligned} f_i(z^{-1}) &\triangleq E\{e_i(k)\{h_{11}(z^{-1})u_1(k) + h_{21}(z^{-1})u_2(k)\}\} / E\{|e_i(k)|^2\}, \\ &= E\{e_i(k)\{h_{11}(z^{-1})e_1(k)/a_1(z^{-1}) + \\ &\quad h_{21}(z^{-1})e_2(k)/a_2(z^{-1})\}\} / E\{|e_i(k)|^2\}, \end{aligned}$$



z^{-1} : one-step delay operator,

a_i ($i = 1, 2$) denotes a prediction error filter (PEF) that generates $u_i(k)$ from $e_i(k)$ such as $a_i = 1 - \{a_{i:0}z^{-1} + \dots + a_{i:m+L}z^{-(m+L+1)}\}$.

Fig. 2 Schematic diagram of a multichannel equalizer to recover the signal $u_1(k)$ from its innovation $e_1(k)$.

The signal $u_1(k)$, which once returns to its innovation $e_1(k)$ through linear predictors w_j , is re-generated through the inverse-PEF, $1/a_1$, that is calculated by the PEF-estimator.

$$= h_{11}(z^{-1})/a_1(z^{-1}). \quad (13)$$

Since the polynomial $h_{11}(z^{-1})$ may also be assumed to be coprime with the inverse PEF $1/a_2(z^{-1})$, the inverse PEF $1/a_1(z^{-1})$ may be calculated as an approximate greatest common divisor of the polynomials $f_1(z^{-1})$ and $1/c_w(z^{-1})$ [18-21].

Interestingly, this procedure also clarifies the correspondence between the innovation $e_1(k)$ and the inverse PEF $1/a_1(z^{-1})$. This additional information may be exploited to re-generate the "original" $u_1(k)$ from the innovation as shown in Fig. 2.

4. CONCLUDING REMARKS

In blind inverse-filtering, linear-recurrent signals may sometimes get whitened. For avoiding this excessive deconvolution, it seems essential to estimate respective PEFs (in other words, AR processes) for generating such linear-recurrent signals so as to remove the contributions of such PEFs from the inverse-filters. This article has demonstrated that respective PEFs that generate linear-recurrent signals may be extractable from convolutive mixtures of these signals in theory.

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