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CONTROL SYSTEMS ANALYSIS VIA BLIND SOURCE DECONVOLUTION

Kenji Sugimoto and Yoshito Kikkawa

Nara Institute of Science and Technology
Graduate School of Information Science
8916-5 Takayama-cho, Ikoma-city, Nara, 630-0192, Japan
{kenji,yoshi-ki}@is.aist-nara.ac.jp

ABSTRACT

This paper studies application of independent component analysis techniques to automatic control engineering. It is often desired in control systems to separate signals which are mixed through some unknown process. Since this process usually has dynamics, blind source deconvolution must be achieved in this case. Under a certain assumption, the paper provides a method to this end, by making use of a time series expansion motivated from control system identification. This method is then applied to simultaneous estimation of both disturbance and system parameter. The result is illustrated by means of a numerical simulation and experiment with a mechanical system.

1. INTRODUCTION

The last decade has witnessed a great deal of progress in Independent Component Analysis (ICA). Today it has a wide range of application areas such as speech, image, telecommunication, and medical signal processing; see, [1, 3, 4, 7, 8], just to name a few.

Automatic control engineering is another area where ICA may be used as a promising tool for signal analysis. In control systems, for example, we sometimes encounter a situation where unknown disturbance signals are mixed into signals to be controlled with unknown mixing mechanism. It is desirable if we can eliminate this disturbance by identifying it. In fact, Principle Component Analysis (PCA) has already been used in this context, and is called statistical process monitoring [10].

In control systems, however, signals are usually processed through dynamical systems, so that we have to deal with time history of the signals. This is a kind of the Blind Source Deconvolution (BSD) problem, which has been extensively studied, and various methods have been proposed to solve it in literature [2, 9, 14].

In this paper, we propose yet another approach to achieve BSD, and apply it to some of the control systems analysis problems. We first introduce the concept of polynomial matrix fractional representation of linear dynamical systems. Then we expand a series of the time signal into a vector form using polynomial matrix coefficients, which enables us to apply a standard ICA algorithm to the system.

In what follows, we confine ourselves to discrete-time signals for simplicity. Generalization to the continuous-time case is straightforward. The time t hence takes integer values. With a slight abuse of language, we use z as the forward time-shift operator, i.e., $zu(t) := u(t+1)$. As usual, I represents the identity matrix with an appropriate size, and A^T denotes the transpose of a matrix A.

The remainder of the paper is organized as follows: After problem formulation for BSD is given in §2, we study two special cases in §3 with numerical simulation. An experiment with a mechanical setup is explained in §4. We conclude the paper in §5.

2. PROBLEM FORMULATION

Let us consider the following dynamical system:

$$\eta(t) = G(z)s(t), \qquad t = 1, 2, \cdots , \tag{1}$$

where $G(z)$ is a discrete-time transfer matrix. We assume that each component $s_1(t), \cdots , s_m(t)$ of an unknown signal vector $s(t)$ is statistically independent, and $\eta(t)$ is an observed signal vector.

To explain the proposed method, let us introduce a special assumption on $G(z)$. To begin with, we assume that $G(z)$ is proper rational, square, invertible, and stable. The following notions are well known in linear system theory [6, 11, 12, 13]:

Fact 1. Any rational matrix $G(z)$ can be represented by means of the polynomial matrix fraction:

$$G(z) = D(z)^{-1}N(z) \tag{2}$$

where $D(z)$ and $N(z)$ are left coprime, and $D(z)$ is row proper (Fig. 1).

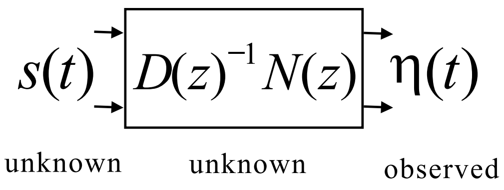

Fig. 1. Dynamical system under consideration.

Definition. $D(z)$ and $N(z)$ are called left coprime if the following condition holds: If there exist polynomial matrices $\tilde{D}(z)$, $\tilde{N}(z)$, and $U(z)$ such that

$$D(z) = U(z)\tilde{D}(z), \text{ and } N(z) = U(z)\tilde{N}(z),$$

then $U(z)$ must be unimodular; i.e., $\det U(z)$ is a constant number.

Definition. A square polynomial matrix $D(z)$ is called row proper if $[D(z)]_R$ is nonsingular, where $[D(z)]_R$ is the constant matrix whose rows are the highest coefficient vectors of the corresponding rows of $D(z)$; i.e.,

$$[D(z)]_R := \begin{pmatrix} {d_1}^0 \\ \vdots \\ {d_m}^0 \end{pmatrix}, \text{ where } D(z) = \begin{pmatrix} d_1(z) \\ \vdots \\ d_m(z) \end{pmatrix},$$

$$d_i(z) = {d_i}^0 z^{\mu_i} + {d_i}^1 z^{\mu_i - 1} + \cdots + {d_i}^{\mu_i}, \quad {d_i}^0 \neq 0.$$

Furthermore, μ_i is called the i-th row degree of $D(z)$.

In order to avoid certain redundancy in the representation (2), left coprimeness and row properness are required. However, there still remains a non-uniqueness in fractional representation as above. In order to guarantee the uniqueness, we further need to introduce a kind of canonical form called the polynomial Echelon form [6]. A detail is omitted in this paper for notational simplicity.

The polynomial matrix fraction given above is closely related to state space representation:

Fact 2. Let (A, B, C, J) be a minimal realization of the transfer matrix $G(z)$ in (1). If (2) is a left coprime factorization and $D(z)$ is row proper, then the row degrees $\mu_1, \cdots, \mu_m$ of $D(z)$ are equal to the observability indices of (A, C), respectively.

Now we are ready to give our fundamental assumption:

A1) The row degrees $\mu_1, \cdots, \mu_m$ are known.

A2) The matrix $[D(z)]_R$ is known. Then, with no loss of generality, we further assume $[D(z)]_R = I$. The other coefficients of $D(z)$ are unknown.

A3) $N(z) = \text{diag}\,(z^{\mu_1}, \cdots, z^{\mu_m})N_0$ for a constant unknown matrix N_0. Denote by ${n_i}^0$ the i-th row of N_0.

Let us rewrite the i/o relation (1) by (2):

$$D(z)\eta(t) = N(z)s(t). \tag{3}$$

Under the assumptions A1), A2), and A3), we can reduce (3) to

$$\eta_i(t) = {n_i}^0 s(t) - {d_i}^1 \eta(t-1) - \cdots - {d_i}^{\mu_i}\eta(t-\mu_i) \tag{4}$$

for $i = 1, \cdots, m$, and $\eta(t) = (\eta_1(t), \cdots, \eta_m(t))^T$. From this equation, we see that the above assumptions correspond to a certain class of systems of auto-regressive (AR) type.

3. MAIN RESULT

In this section, we propose a scheme for BSD under the assumption in the previous section. For the sake of notational simplicity, two special cases are explained. Furthermore, to verify the effectiveness of the proposed scheme, a numerical example is given for each case.

3.1. Simultaneous estimation of system parameter and disturbance

3.1.1. Main result

If μ_i's are not necessarily equal and

$$D(z) = \text{diag}(d_{11}(z), \cdots, d_{mm}(z)), \tag{5}$$

then the equation (4) becomes

$$\eta_i(t) = {n_i}^0 s(t) - {d_{ii}}^1 \eta_i(t-1) - \cdots - {d_{ii}}^{\mu_i}\eta_i(t-\mu_i), \tag{6}$$

where

$$d_{ii}(z) = z^{\mu_i} + {d_{ii}}^1 z^{\mu_i - 1} + \cdots + {d_{ii}}^{\mu_i}.$$

From (6), let us consider the linear expansion

$$\begin{pmatrix} \eta_1(t-1) \\ \vdots \\ \eta_1(t-\mu_1) \\ \vdots \\ \eta_m(t-1) \\ \vdots \\ \eta_m(t-\mu_m) \\ \eta(t) \end{pmatrix} = \begin{pmatrix} I & 0 \\ \Theta & N_0 \end{pmatrix} \begin{pmatrix} \eta_1(t-1) \\ \vdots \\ \eta_1(t-\mu_1) \\ \vdots \\ \eta_m(t-1) \\ \vdots \\ \eta_m(t-\mu_m) \\ s(t) \end{pmatrix} \tag{7}$$

where Θ is a coefficient matrix having a block diagonal structure. Since $\eta_1(t-1), \cdots, \eta_m(t-\mu_m)$ appear both in the left- and right-hand sides, it is observed that the coefficient matrix in the equation (7) has fixed elements. From this expansion, we can estimate non-fixed elements and the source signal vector $s(t)$ by slightly modifying a standard ICA algorithms.

Now we proceed to applying the above approach to simultaneous estimation of disturbance and system parameter. Automatic control systems are often affected from their

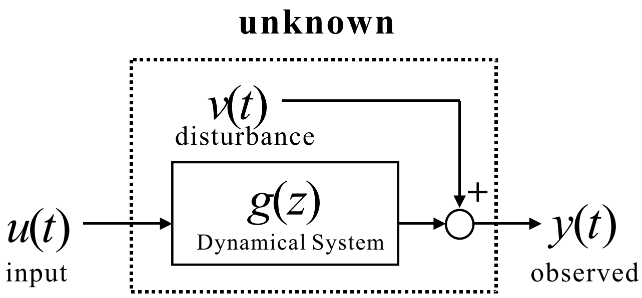

Fig. 2. Disturbance estimation.

environment. For example, chemical process is affected by the outside temperature, and mechanical systems are exposed to small vibration from the earth. Such unexpected effect is called "disturbance", and is mixed with the controlled signal with an unknown coefficients.

Let us consider the next scalar system (Fig. 2).

$$y(t) = g(z)u(t) + v(t) \tag{8}$$

$$g(z) = \frac{b(z)}{a(z)} = \frac{b_r z^{n-r} + \cdots + b_n}{z^n + a_1 z^{n-1} + \cdots + a_n} \tag{9}$$

where $v(t)$ is an unknown disturbance, and the parameters $a_1, \cdots, a_n; b_r, \cdots, b_n$ are also assumed to be unknown. We assume that $g(z)$ is stable and of minimal phase, the order n and the relative degree r are known, and hence $b_r \neq 0$. Here, we can regard disturbance and signals in the system as statistically independent, because they are caused by different, irrelevant factors. We can see that this problem is included in (7) by defining

$$D(z) = \begin{pmatrix} b(z)/b_r & 0 \\ 0 & a(z) \end{pmatrix}, N(z) = \begin{pmatrix} 1/b_r & 0 \\ 1 & 1 \end{pmatrix}, \tag{10}$$

and

$$s(t) = \begin{pmatrix} b(z)u(t) \\ a(z)v(t) \end{pmatrix}. \tag{11}$$

Moreover, if the fact that N_0 has the special lower triangular structure is known in advance, (8) can be represented as

$$y(t) = \theta^T \omega(t) + \epsilon(t), \tag{12}$$

where $\epsilon(t) := z^{-n}a(z)v(t)$ and

$$\theta = \begin{pmatrix} -a_1 \\ \vdots \\ -a_n \\ b_r \\ \vdots \\ b_n \end{pmatrix}, \quad \omega(t) = \begin{pmatrix} y(t-1) \\ \vdots \\ y(t-n) \\ u(t-r) \\ \vdots \\ u(t-n) \end{pmatrix}. \tag{13}$$

Then this problem can be reduced as:

$$\begin{pmatrix} \omega(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} I & 0 \\ \theta^T & 1 \end{pmatrix} \begin{pmatrix} \omega(t) \\ \epsilon(t) \end{pmatrix} \tag{14}$$

3.1.2. Numerical example

The simulation is performed using MATLAB. For ICA algorithm, we adopted the method by [8], and modified it according to the idea in [5] so that we can treat the fixed block structure.

Let us consider the first-order system:

$$g(z) = \frac{1}{z + 0.8} \tag{15}$$

We used the M-sequence as an input signal $u(t)$, and a sequence of normal random numbers as the disturbance signal $v(t)$. The number of samples for the signals is 512.

We first generated an output signal from the given input signal. Fig. 3 shows the input and the output signals. Then we estimated source signal (disturbance) only by this output signal by means of ICA algorithm. Fig. 4 shows the result of estimation. Upper plot represents the original disturbance and lower plot represents the estimated disturbance. From this figure, we can see that our scheme separates mixed signals successfully.

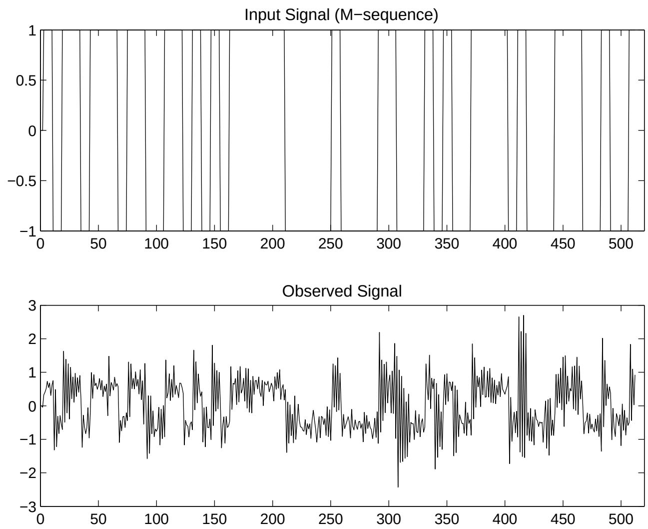

Fig. 3. Input and observed signals.

Next, we estimated system parameters. The matrix to be estimated is below.

$$\begin{pmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ -0.8 & 1.0 & 1.0 \end{pmatrix} \tag{16}$$

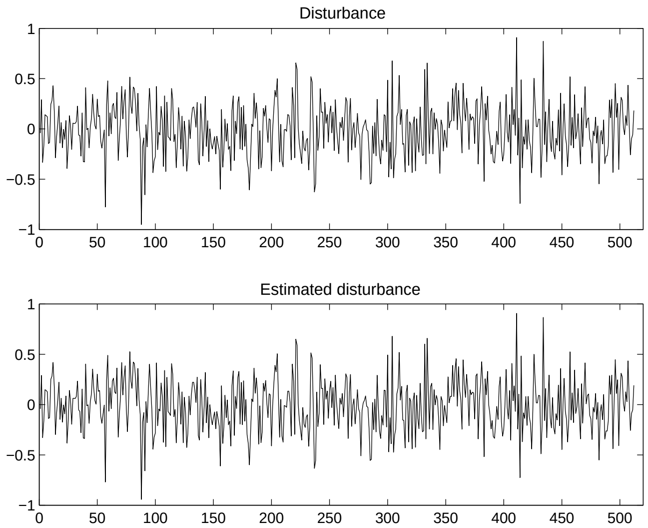

Fig. 4. Reconstruction of disturbance.

The result of the parameter estimation is below. The true value -0.8 is estimated as -0.8022. Since the relative error of estimated system parameter is 0.28%, the result is satisfactory enough.

$$\begin{pmatrix} 0.9981 & 0.0011 & -0.0000 \\ 0.0017 & 0.9981 & 0.0000 \\ -0.8022 & 0.9939 & 1.000 \end{pmatrix} \tag{17}$$

3.2. Separation of signals which are mixed by a dynamical system

3.2.1. *Main result*

If $\mu_1 = \cdots = \mu_m (=: \mu)$, then we have

$$D(z) = Iz^{\mu} + D_1 z^{\mu-1} + \cdots + D_{\mu},$$

and (4) becomes

$$\eta(t) = N_0 s(t) - D_1 \eta(t-1) - \cdots - D_{\mu}\eta(t-\mu).$$

In this case, we obtain the linear expansion

$$\begin{pmatrix} \eta(t-1) \\ \vdots \\ \eta(t-\mu) \\ \eta(t) \end{pmatrix} = \begin{pmatrix} I & 0 \\ \Theta & N_0 \end{pmatrix} \begin{pmatrix} \eta(t-1) \\ \vdots \\ \eta(t-\mu) \\ s(t) \end{pmatrix}. \tag{18}$$

where

$$\Theta := \begin{pmatrix} -D_1 & \cdots & -D_{\mu} \end{pmatrix}. \tag{19}$$

From this expansion, we can separate observed signals by the existing ICA algorithm again (Fig. 5).

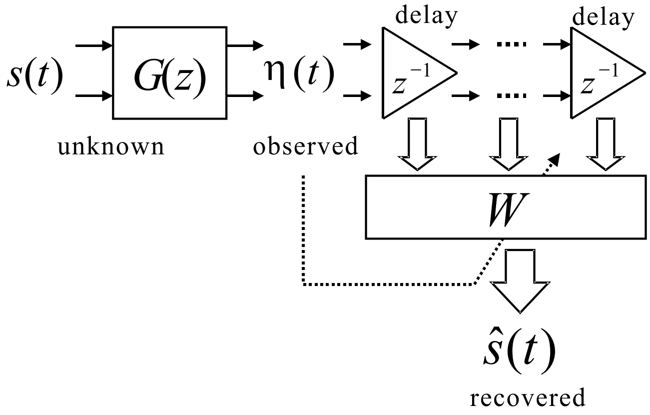

Fig. 5. Proposed Scheme for Reconstruction.

3.2.2. *Numerical example*

In this subsection, FastICA algorithm is used to separate the mixed signals. The MATLAB code for the FastICA was downloaded from the home-page provided by the authors of [4] and [5].

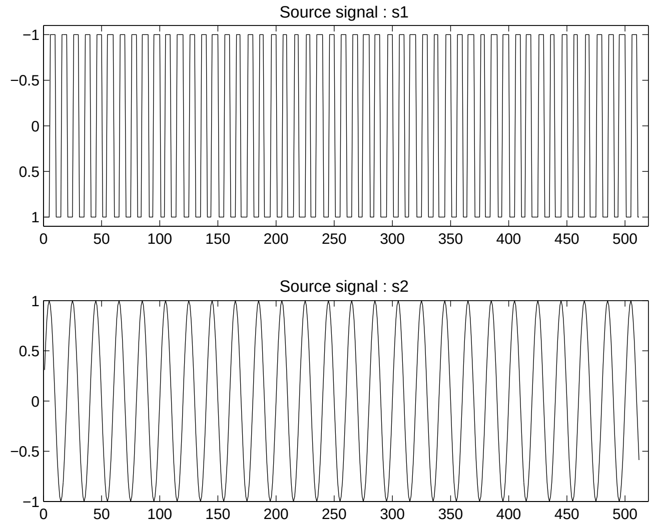

Fig. 6. Source Signals.

As source signals, we take rectangular (s_1) and sine (s_2) waves, both of which have unit amplitude (Fig. 6). These signals are processed by the second-order dynamical system (Fig. 7):

$$D(z) = Iz^2 + D_1 z + D_2, \quad D_1 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.3 \end{pmatrix},$$

$$D_2 = \begin{pmatrix} 0.02 & 0 \\ 0 & 0.03 \end{pmatrix}, \quad N_0 = \begin{pmatrix} 1 & 0.2 \\ 0.2 & 1 \end{pmatrix}. \tag{20}$$

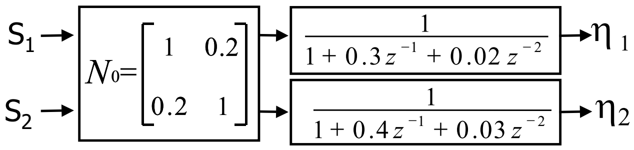

Fig. 7. Mixing model in § 3.2 .

Then the output signal $\eta(t) := (\eta_1,\ \eta_2)^T$ is observed as shown in Fig. 8. Note that in this case, $\mu = 2$ in (18). We therefore apply the six signals $\eta_1(t), \eta_2(t), \eta_1(t-1), \eta_2(t-1), \eta_1(t-2), \eta_2(t-2)$ to the FastICA program.

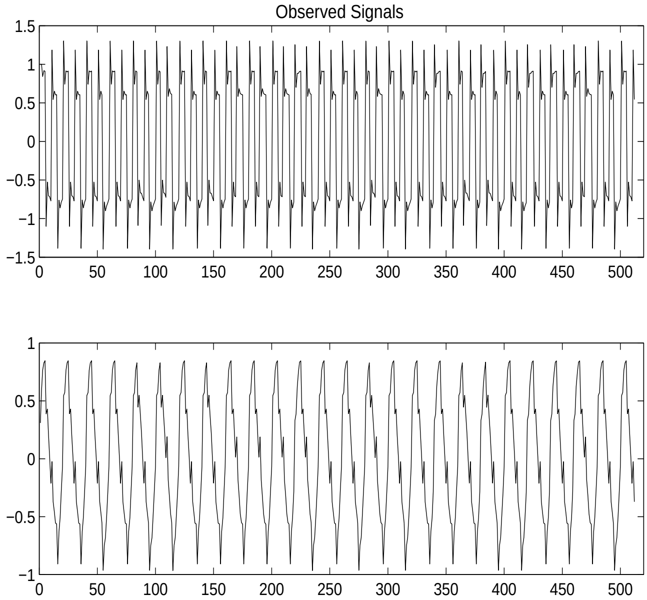

Fig. 8. Mixed signals

The FastICA program has returned six signals as a result (Fig. 9), but the source signals to be extracted are just two signals (s_1 and s_2). We find that they are the upper two plots of the figure, and that our scheme separates observed signals successfully.

In order to evaluate this separation in more detail, let us examine the matrix N_0. After applying some normalization, we obtain the estimated values as:

$$N_0 = \begin{pmatrix} 1.0026 & 0.2010 \\ 0.2132 & 1.0052 \end{pmatrix}$$

These values coincide with the true values in (20) quite well, and the mean relative error is calculated as 1.97 % .

4. EXPERIMENT

In this section, we introduce an experimental setup in order to check effectiveness of the proposed scheme in the real world. Our idea is illustrated in Fig. 10.

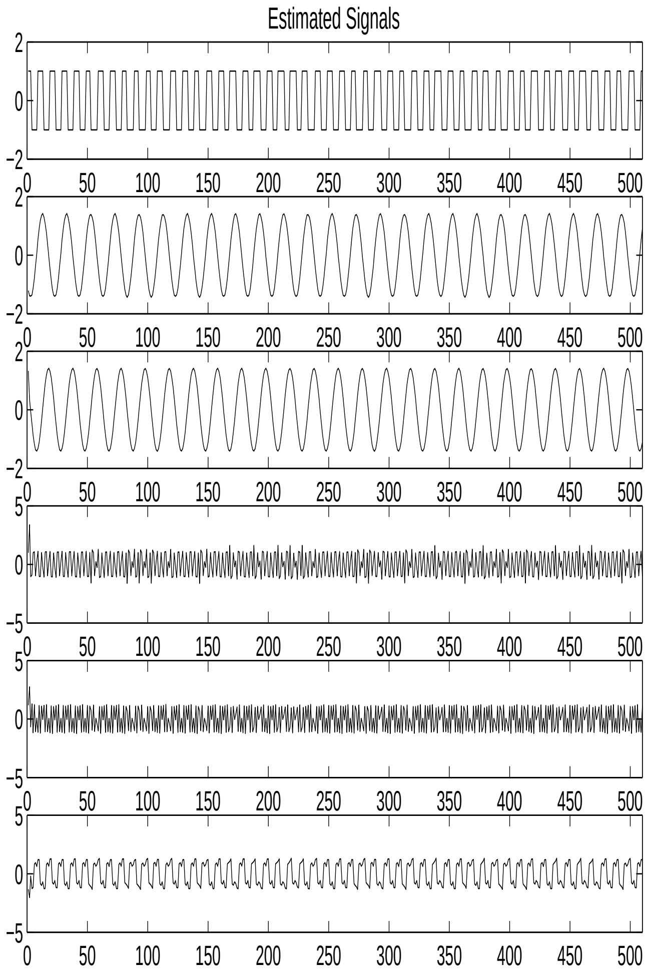

Fig. 9. Signals estimated by FastICA

We detect oscillation of this device by measuring the displacement of the two markers. Two source signals (vibration and disturbance) go through the dynamical system (spring balance) and are mixed on the pole. In order to meet the assumption in §3.2, we assume that the position of those markers is unknown, and that the physical parameter of both spring balances is also unknown.

As a preliminary experiment, we have applied FastICA to the case *without the springs*, namely the static mixture case with the coefficients determined by the positions of the markers. Two sine waves of different periods have been used as source signals.

The recovered signals by our scheme are shown in Fig. 11. Below, the normalized separation matrix is given in the left hand side, while the right hand side is the true value from

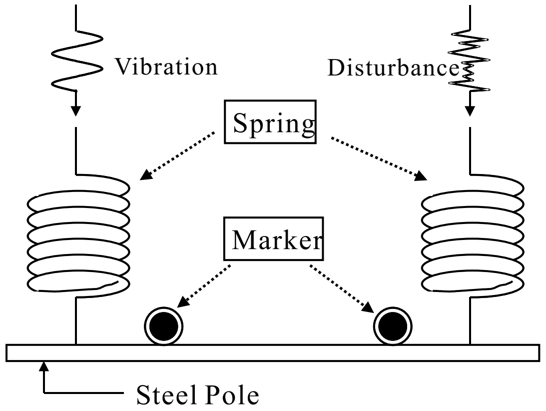

Fig. 10. Experimental setup

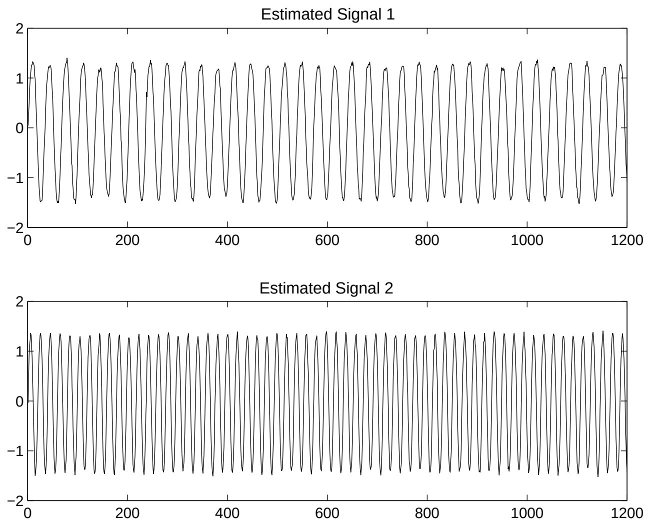

Fig. 11. Estimated signals of preliminary experiment

physical measurement.

$$\begin{pmatrix} 0.8851 & -0.4654 \\ -0.5357 & 0.8444 \end{pmatrix}, \quad \begin{pmatrix} 0.8721 & -0.4894 \\ -0.5316 & 0.8470 \end{pmatrix}.$$

The relative error is calculated as 1.87%.

5. CONCLUSION

In this paper, we have proposed an ICA scheme for separating signals mixed by a class of dynamical systems, and studied its application to control engineering.

Under the assumption that the dynamical system belongs to a certain class of AR type, we have reduced BSD problem into BSS problem by using polynomial matrices. Some numerical examples have shown effectiveness of this scheme. We belive that it is important to solve such BSD problems, since this is useful for control engineering as well as other applications such as speech with echo.

We also introduced an experimental setup in the last section. This experiment setup can be considered as a simple model of various real worlds' phenomena such as oscillations of a floor caused by constructions or earthquake.

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