EXPERIMENTAL STUDIES ON DOA ESTIMATION BASED ON BLIND SIGNAL SEPARATION AND ARRAY SIGNAL PROCESSING [1]

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ABSTRACT

In many applications, the direction-of-arrival (DOA) estimation problem is of great interest. After blind separation of the signals incident onto the receiving sensor array, the DOAs can be estimated for each source individually. Non-Gaussian signals with negative kurtosis can be automatically captured by the constant modulus (CM) array, which is the most striking blind beamforming algorithm and widely discussed. However, very little experimental studies have been conducted so far. In this paper, after proposing a new method, we analyze the DOA estimation performance of the multistage CM array via computer simulations and water tank experiments, and compare it with that of other DOA estimation algorithms including 'non-blind' and 'blind' algorithms. The multistage CM array shows better results in all considered situations.

1. INTRODUCTION

Direction-of-arrival (DOA) estimation of multiple signals impinging on a sensor array is a well-studied problem in array signal processing. Traditional methods, such as MUSIC [1] and ESPRIT [2], exploit knowledge of the array geometry without using information of the signal itself. However, the assumption of a known array response is seldom satisfied in practice, which makes the DOA estimation performance of these traditional methods degrade significantly [3]. Recently, some methods are studied to recover the information of signals incident onto the array by exploiting their properties only, such as high-order statistics [4], cyclostationarity [5] or constant modulus (CM) property [6]. Beamforming based on these methods are called blind beamforming, since no knowledge of the array geometry is required. After blind separation of the signals, the DOA estimation problem is decoupled and can be done for each source individually [7]. Blind beamforming methods are more robust to array manifold errors due to the extra information they use.

Since the pioneering work of Treichler and Agee [8], it is known that the CM property is a strong property that, by itself, is sufficient for source separation. Such a scheme is proposed in [9], where the CM signals are successively separated using the so-called CM array. The CM array has fast convergence properties and low computational complexity. Moreover, the non-Gaussian signals with negative kurtosis can be automatically captured by the CM array [10]. One stage of the CM array consists of a conventional weight-and-sum adaptive beamformer whose weights are updated by the constant modulus algorithm (CMA) [8] without using any training signal. An adaptive signal canceller is included in each stage to remove a captured source from the input to next stage. In this way, several narrowband source signals can be sequentially extracted at successive stages of the system. If the array geometry is known, the DOAs can be estimated [11,12]. Signal separation and reconstruction do not require knowledge of the array configuration.

Knowing that the multistage CM array is a good algorithm, it becomes interesting to study the DOA estimation performance in practical situations. Here,

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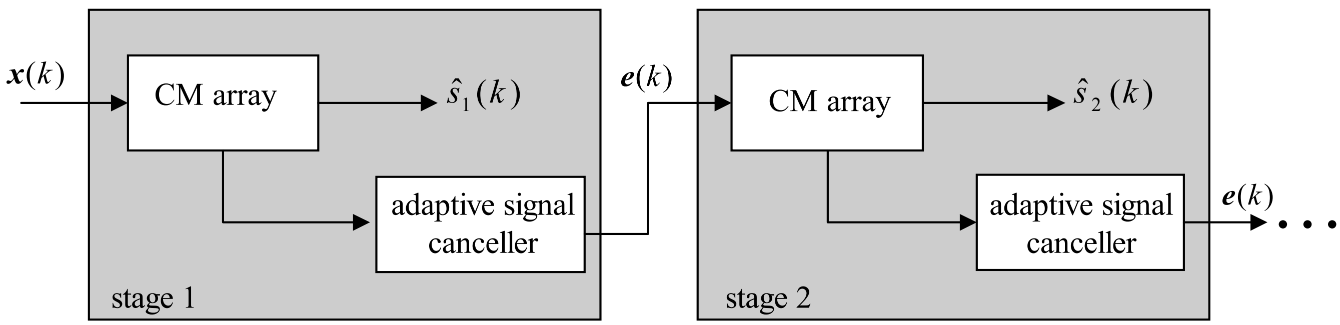


Fig. 1 the cascade multistage CM array

using the results of the computer simulations and water tank experiments, our aim is to demonstrate the DOA estimation performance of the CM array for multiple signals and compare it with other DOA estimation algorithms which include 'non-blind' and 'blind' algorithm.

This paper is organized as follows. Section 2 defines the data model. In section 3, we present a brief overview of multistage CM array. A new DOA estimation method is derived in section 4. Computer simulations and water tank experiments are present in section 5 to illustrate the DOA estimation performance and robustness for array model errors. Finally, concluding remarks are given in section 6.

2. DATA MODEL

Consider a scenario in which several narrowband sources are contaminated by additive noise. Assume that there are d non-Gaussian signals $\{s_i(k)\}_{i=1}^{d}$ with negative kurtosis impinging on an array of M sensors from directions $\{\theta_i\}_{i=1}^{d}$. The array may have arbitrary and unknown response and geometry. Thus, the array input $\boldsymbol{x}(k)=[x_1(k),\cdots,x_M(k)]^T$ can be written as

$$\boldsymbol{x}(k)=\boldsymbol{a}(\theta_1)s_1(k)+\cdots+\boldsymbol{a}(\theta_d)s_d(k)+\boldsymbol{n}(k) \qquad (1)$$

or

$$\boldsymbol{x}(k)=\boldsymbol{A}\boldsymbol{s}(k)+\boldsymbol{n}(k) \qquad (2)$$

where superscript T denotes the transpose operation. $\boldsymbol{s}(k)=[s_1(k),\cdots,s_d(k)]^T$ is the source signal vector, $\boldsymbol{n}(k)=[n_1(k),\cdots,n_M(k)]^T$ is an additive Gaussian noise vector with unknown variance. $\boldsymbol{s}(k)$ and $\boldsymbol{n}(k)$ are temporally uncorrelated. $\boldsymbol{A}=[\boldsymbol{a}(\theta_1)\ \cdots\ \boldsymbol{a}(\theta_d)]$ is referred to as the array manifold, which includes the information about the array geometry responses and the directions of the impinging sources. Its column vector $\boldsymbol{a}(\theta_i)$ is the array response to the ith source, and called the direction vector.

If the impinging signals are coherent, then we have

$$\boldsymbol{A}\boldsymbol{s}(k)=\sum_{i=1}^{d}\boldsymbol{a}(\theta_i)\beta_i s_1(k) \qquad (3)$$

where β_i is the complex scaling factor of ith signal relative to the first (or reference) signal. Now we have the generalized direction vector $\boldsymbol{b}(\Theta)$

$$\boldsymbol{b}(\Theta)=\sum_{i=1}^{d}\beta_i\boldsymbol{a}(\theta_i) \qquad (4)$$

which is a linear combination of the direction vectors $\{\boldsymbol{a}(\theta_i)\}_{i=1}^{d}$ for all coherent signals. This result shows that the case of coherent signals is equivalent to the case of statistically independent sources with a modified direction vector.

The sample correlation matrix $\boldsymbol{R}_x=\mathrm{E}[\boldsymbol{x}(k)\boldsymbol{x}^H(k)]$ can also be expressed in terms of its eigenvalues and eigenvectors. If the eigenvalues of $\boldsymbol{R}_x$ are represented by the diagonal elements of $\boldsymbol{D}$ and the corresponding eigenvectors by column vectors of $\boldsymbol{F}$, a pre-whitening matrix can be defined as

$$\boldsymbol{V}=\boldsymbol{D}^{-\frac{1}{2}}\boldsymbol{F}^H \qquad (5)$$

Let $\tilde{\boldsymbol{X}}=\boldsymbol{V}\boldsymbol{X}$ and we pre-whiten the original array sample signals. All processing presented in this paper was based on the pre-whitened data.

3. CASCADE MULTISTAGE CM ARRAY

A model of the cascade multistage CM array is shown in Fig.1. According to the CMA, the cost function of CM array can be written as:

$$J(k)=E[||y(k)|-1|^2]\to\min \qquad (6)$$

Using the steepest–descent method and replacing the gradient vectors with their instantaneous estimates, the iterative procedure of the CM array can be described by the following equations:

$$\begin{cases} y(k) = \boldsymbol{w}^H(k)\boldsymbol{x}(k) \\ e_c(k) = y(k)/|y(k)| - y(k) \\ \boldsymbol{w}(k+1) = \boldsymbol{w}(k) + \mu_{\text{CMA}}\boldsymbol{x}(k)e_c^*(k) \end{cases} \quad (7)$$

which is a realization of the LMS algorithm with a desired signal $y(k)/|y(k)|$. The step size $\mu_{\text{CMA}} > 0$ controls the convergence rate of CM array.

The adaptive signal canceller can directly use the complex LMS algorithm, and be updated by

$$\begin{cases} y(k) = \boldsymbol{w}^H(k)\boldsymbol{x}(k) \\ \boldsymbol{e}(k) = \boldsymbol{x}(k) - \boldsymbol{u}(k)y(k) \\ \boldsymbol{u}(k+1) = \boldsymbol{u}(k) + 2\mu_{\text{LMS}}y^*(k)\boldsymbol{e}(k) \end{cases} \quad (8)$$

The step size is bounded by $0 < \mu_{\text{LMS}} < 1/\sigma_y^2$, where $\sigma_y^2 = E[|y(k)|^2]$ is the variance of CM array output at the current stage. Thus, the convergence properties of the canceller weights depend on weights of the CM array, whereas the CM array weights are independent of those of the adaptive canceller.

4. DOA ESTIMATION

Blindly estimating the array manifold $\hat{\boldsymbol{A}}$, we can estimate the DOA of each source by projecting each column vector $\hat{\boldsymbol{a}}(\theta_i)$ of $\hat{\boldsymbol{A}}$ onto the array manifold. $\boldsymbol{A}$ can be obtained by computing the pseudo-inverse matrix of weight matrix $\boldsymbol{W}$ constructed in blind beamforming. In this paper, based on the blind source reconstruction, we present a new estimation method with least-squares estimation. Having the estimated signals $\hat{\boldsymbol{S}} = [\hat{\boldsymbol{s}}_1, \cdots, \hat{\boldsymbol{s}}_d]^T$ at hand, $\boldsymbol{A}$ can be estimated by

$$\hat{\boldsymbol{A}} = \arg\min_{\boldsymbol{A}}\{\|\boldsymbol{X} - \boldsymbol{A}\hat{\boldsymbol{S}}\|_F^2\} \quad (9)$$

then we can obtain the optimum solution of $\boldsymbol{A}$

$$\hat{\boldsymbol{A}} = \boldsymbol{X}\hat{\boldsymbol{S}}^H\left(\hat{\boldsymbol{S}}\hat{\boldsymbol{S}}^H\right)^{-1} \quad (10)$$

By the column vectors $\{\hat{\boldsymbol{a}}(\theta_i)\}_{i=1}^d$ of $\boldsymbol{A}$, the DOA can be estimated by

$$\theta_i = \arg\max_{\theta}\left|\hat{\boldsymbol{a}}(\theta_i)^H\boldsymbol{a}(\theta)\right|^2, \qquad i = 1, \cdots d \quad (11)$$

where $\boldsymbol{a}(\theta)$ is the scanning vector, and θ varies within the whole searching space.

5. EXPERIMENTS RESULTS

Computer simulations and water tank experiments were conducted to study the source separation and DOA estimation performance of the multistage CM array. A uniform linear array (ULA) was assumed so as to be consistent with the theoretical work in earlier sections, and the results are presented in this section.

In this section, we compare the DOA estimation performance of four algorithms including conventional beamforming (CBF), MUSIC, cumulant-based blind beamforming (CUM), and the multistage CM array. The former two non-blind algorithms estimate DOAs based on the spatial spectra, and the other two blind beamforming algorithms do it by projecting each direction vector $\hat{\boldsymbol{a}}(\theta_i)$ onto the array manifold.

In all experiments, the ith CM array weight was initialized to $\boldsymbol{w}_i(0) = \boldsymbol{i}$ to avoid the CM arrays converge to the same signal. The ith element of the unit vector $\boldsymbol{i}$ was 1 and all others are zero. The step size $\mu_{\text{CMA}} = 0.005$. Signal canceller weights were initialized as $\boldsymbol{u}(0) = \boldsymbol{0}$ and the initial step size $\mu_{\text{LMS}} = 0.02$. The results were averaged over 100 runs, with a sample length of 1024 in each run. The order of the multistage CM array is equal to the number of sources.

5.1. Computer simulations

In the computer simulations, an uniform linear array (ULA) of 16 identical isotropic sensors spaced a half wavelength apart was used. Assume that there were 2 independent narrowband signals impinging on the array and the additive noise was spatial white Gaussian noise. Beampatterns of each stage of the two-stage CM array is showed in Fig.2.

Fig.3 shows the DOA estimate biases and standard deviations of source 2 at varying source separation from 2 ° to 20 °, where the input SNR of each source is 10dB. Results from 4 beamforming methods are

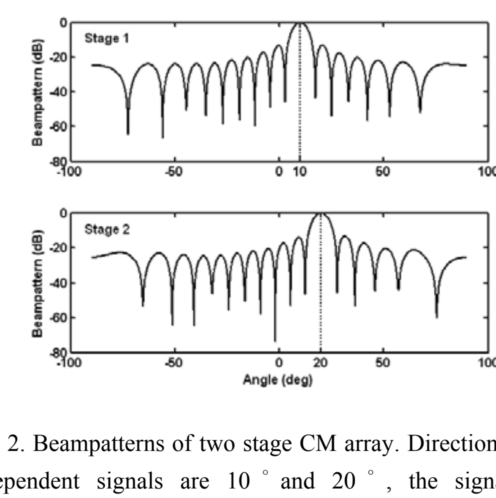


Fig. 2. Beampatterns of two stage CM array. Directions of two independent signals are 10 ° and 20 °, the signal input SNR=[10dB, 5dB].

compared in these plots. The Rayleigh beamwidth of such a 16-sensor ULA is 7°, i.e. CBF will fail to resolve 2 sources when their angular separation is smaller than 7°. MUSIC demonstrates high resolution ability in solving uncorrelated sources at small separations. From the plots, it can be seen that blind beamforming algorithms are also effective when 2 sources are closely placed, as long as the sources are blindly separated.

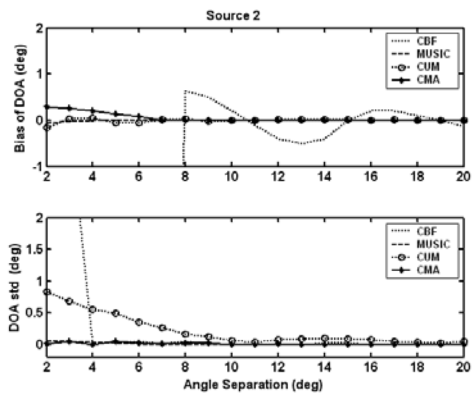


Fig. 3. Performance of DOA estimation for the second source at varying angular separations.

5.2. Water tank experiments

Water tank experiments were conducted in the anechoic water tank of Northwestern Polytechnical University, whose size is $20\text{m} \times 8\text{m} \times 7\text{m}$. A 16-element ULA with 10cm interelement spacing was used as the receiving array. Two sources were placed at distances from the array. During the experiments, the position of one source was fixed while the other one was placed at different positions to generate signals from varying directions. In experiment one, two sources were excited by two different sinusoids at frequencies of 10kHz and 9kHz to simulate two uncorrelated sources. In experiment two, two sources were excited by the same signal generator at frequency of 10kHz to produce two coherent signals. The sample frequency was 51.2kHz through the experiments.

The directions of both sources to the normal of the receiving array in three different situations were given in Table 1. In off-line data processing, data from total 16 sensors and those only from the central 8 sensors were used, respectively, to compare the resolvability of 4 beamforming methods. Ratios of angular separations to the respective Rayleigh beamwidths are also shown in the same Table.

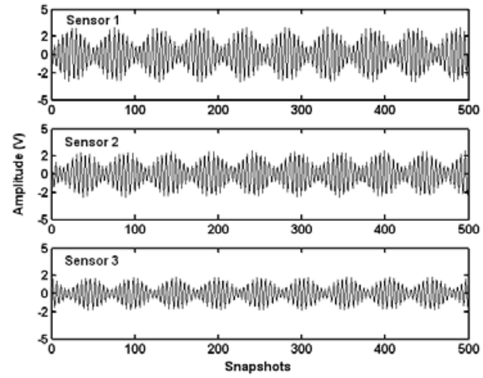


Fig. 4. Output sample signal amplitude of the first, second and third sensor.

Fixing two sources in -22.5° and -9.1°, Fig. 4 shows the signal amplitudes at three different sensors.Inconsistency in the gains is evident. Other errors including sensor position error and phase error were also present. To compare the robustness of different algorithms, no array calibration was conducted before data processing.

Table 1. the ratio of angle separation to the beamwidth at frequency of 10kHz

	16-sensors	8-sensors
Beamwidth	4.5°	9.1°
(−22.5°,−9.1°)	2.98	1.47
(−22.5°,−15.4°)	1.58	0.78
(−22.5°,−17.8°)	1.04	0.52

The first experiment was designed to examine the performance for uncorrelated sources and two different frequencies were used. The spectrum of output at the third sensor is illustrated in Fig.5. Fig.6 (a) and (b) present the beampatterns of the two stages of the CM array and the spectra of reconstructed signals.

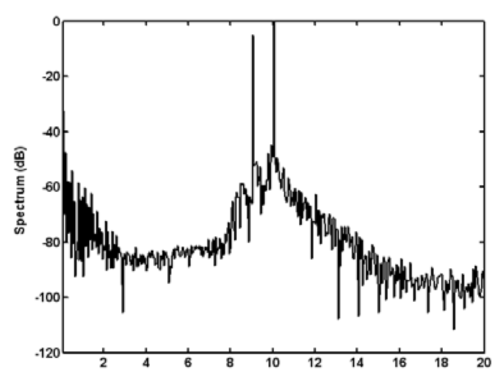


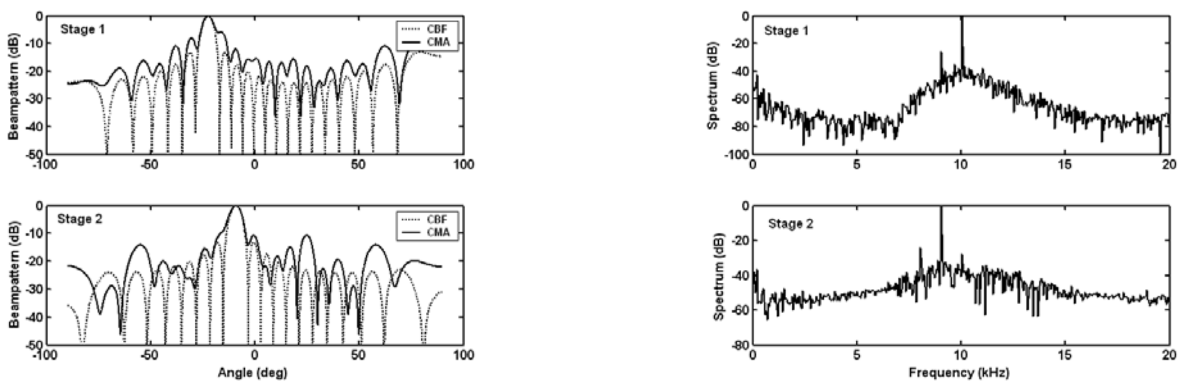
Fig. 5. The output sample signal Spectrum of the third sensor

Table 2. DOA Estimation Mean of Different Algorithm with the 16-sensors (°)

	(−22.5，−9.1)	(−22.5，−15.4)	(−22.5，−17.8)
CBF	(−22.40，−9.30)	(−22.50，−16.00)	(−22.60，−18.70)
MUSIC	(−22.40，−9.30)	(−22.40，−15.40)	(−22.20，−19.40)
CUM	(−22.50，−9.20)	(−22.50，−15.10)	(−22.60，−19.00)
CMA	(−22.50，−9.10)	(−22.54，−15.00)	(−22.60，−18.95)

Table 3. DOA Estimation Mean of Different Algorithm with the 8-sensors(°)

	(−22.5，−9.1)	(−22.5，−15.4)	(−22.5，−17.8)
CBF	(−22.50，−9.20)	(−22.00，18.00)	(−21.40，17.30)
MUSIC	(−22.20，−9.60)	(−21.90，−14.16)	(−21.80，17.40)
CUM	(−22.34，−9.70)	(−22.50，−14.20)	(−22.60，−18.40)
CMA	(−22.40，−9.71)	(−22.47，−14.00)	(−22.50，−17.92)

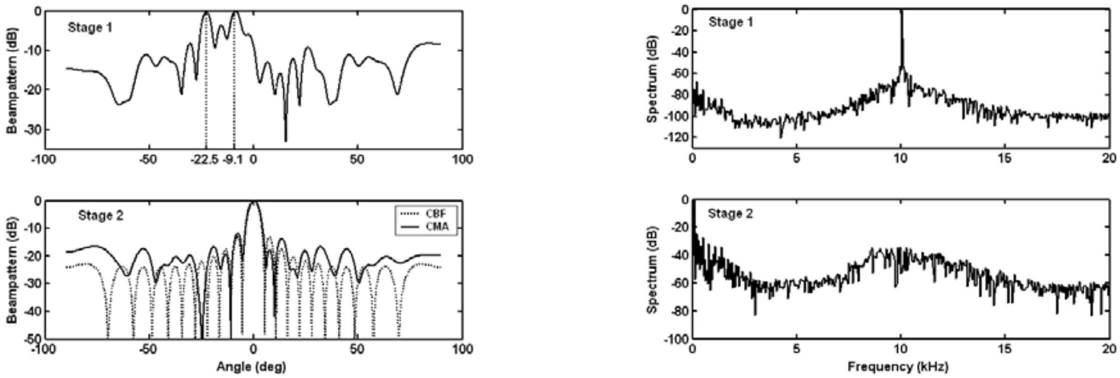


(a) (b)

Fig. 6. 16-sensors ULA receive two independent signals from directions $(-22.5^{\circ}, -9.1^{\circ})$. (a) Beampattern of each stage CM array, and (b) Spectra of CM array outputs.

Results obtained from ULAs with different numbers of sensors are given in Tables 2 and 3. It can be seen that CBF was unable to distinguish two sources within one Rayleigh beamwidth. Due to system errors in the receiving array, MUSIC was ineffective at small source separations. On the other hand, the blind algorithms perform well in all situations. And also, the DOA estimation errors of the multistage CM array are the smallest among four algorithms. The water tank experiments results show that the resolvability of the CM array is beyond the Rayleigh limit and also the CM array is more robust to the model errors of array geometry.

To examine the ability of the CM array to solve two strongly correlated sources, one sinusoid at 10kHz was used to excite both transmitters. Since MUSIC is known to be unable for such a situation, results from the CM array are compared with those from CBF in Fig.7(a). The output spectra of each CM array stage is presented in Fig.7(b). From the plots, we find that the first stage formed two peaks in the true directions of two sources. This is because that the two strongly correlated sources were inseparable within the CM array and was removed by the remove completely by the adaptive signal canceller in the first stage. Without any directional input, the second stage CM array converged to an uniformly weighted and formed a beam in the normal direction (i.e. pointing to 0°). Although the CM array cannot separate highly correlated signals, the estimated generalized direction



(a) (b)

Fig. 7. 16-sensors ULA receive two coherent signals from directions $(-22.5^\circ, -9.1^\circ)$. (a) Beampattern of each CM array stage, and (b) Spectra of CM array outputs.

vector, as described in (4), includes all direction information about the input signals, which should have other ways to resolve them.

6. CONCLUSIONS

Combined with blind signal separation, the DOAs can be estimated at the same time when the sources are recovered. In this paper, we analyzed the DOA estimation performance of the multistage CM array by means of computer simulations and water tank experiments, and compared it with that of three methods, including 'non-blind' and 'blind' algorithms. The results show that the multistage CM array could correctly estimate the DOAs in the cases of both uncorrelated and highly correlated sources, beyond the Rayleigh limit. Also, the CM array is more robust to model errors than some other algorithms.

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