A Multi-Target Track-Before-Detect Particle Filter Using Superpositional Data in Non-Gaussian Noise

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Abstract—We propose a novel method for multi-target track-before-detect using superpositional sensor signals, which depend on the sum of the contributions of all targets. Such superpositional sensor signals arise in diverse domains, such as radio-frequency (RF) tomography, wireless communications, and array signal processing. The proposed method can 1) operate on superpositional sensor signals involving nonlinear target contributions of general functional forms and non-Gaussian observation/process noise to 2) track an unknown, time-varying number of targets 3) without knowing their initial states 4) in an online manner. We conducted a simulation involving superpositional sensor signals in the context of RF tomography. The proposed method outperformed the state-of-the-art superpositional sensor signals in terms of the optimal subpattern assignment (OSPA) metric by a factor of approximately two to four.

Index Terms—Multi-target tracking (MTT), track-before-detect, superpositional sensor signals, particle filter, birth/death process.

I. INTRODUCTION

Multi-target tracking (MTT) aims to estimate time-varying states of multiple targets jointly from available observations, where these states typically include kinematic states (e.g., the position/velocity/acceleration). MTT constitutes one of the most active areas in statistical signal processing, and myriad methods have been proposed. These methods encompass multiple hypothesis tracking (MHT) [1], joint probabilistic data association (JPDA) [2], probabilistic multitarget tracking (PMHT) [3], random finite sets (RFS) [4]–[7], multitarget particle filters [8], [9], and sequential Markov chain Monte Carlo (MCMC) [10].

The conventional approach to MTT is based on a two-step procedure, consisting of detection and tracking steps. In the detection step, sensor signals are preprocessed for detection by, e.g., thresholding, so that each detection corresponds to a single target or a clutter. These detections are fed into the subsequent tracking step. In such a procedure, tracking performance heavily depends on detection performance. Unfortunately, the latter degrades severely under adverse conditions, e.g., at a low signal-to-noise ratio (SNR), and thus, so does the former. Note that, in this approach, information contained in observed time series is not being fully exploited for detection, because it is performed based solely on observations at the current time step without reference to past time steps.

Under adverse conditions, it is significantly advantageous to operate directly on raw sensor signals for joint detection and tracking, which is known in the literature track-before-detect. Compared to the two-step procedure, this approach can exploit information from not only the current but also past time steps in performing detection, leading to better robustness under adverse conditions.

Salmon et al. [11] proposed a track-before-detect method for at most one target in the framework of recursive Bayesian estimation. Since then several authors have considered multi-target extensions of track-before-detect. Kreucher et al. and Vo et al. focused on a restricted class of sensor signals, where targets contribute in a binary [12] or a disjoint [13] manner. Mahler [14], [15] derived a RFS-based filter for multi-target track-before-detect using superpositional sensor signals, which depend on the sum of general nonlinear target contributions. This method is called a superpositional cardinalized probability hypothesis density filter (Σ-CPHD) filter [15]. Moreover, Nannuru et al. [16] developed a tractable approximate implementation of the Σ-CPHD filter based on the particle filter, but it is limited to additive Gaussian observation noise. Boers et al. [17] and Lepoutre et al. [18] considered superpositional sensor signals in the presence of unknown target amplitudes, but again this is limited to additive Gaussian observation noise. Orton et al. [19] proposed a particle filter for multi-target track-before-detect using superpositional sensor signals, but this method is not applicable to the case of an unknown, time-varying number of targets as it is.

Here we propose a novel method for multi-target track-before-detect using superpositional sensor signals. It can 1) operate on superpositional sensor signals involving nonlinear target contributions of general functional forms and non-Gaussian observation/process noise to 2) track an unknown, time-varying number of targets 3) without knowing their initial states 4) in an online manner. The proposed method is a multi-target extension of Salmon et al.’s single-target track-before-detect particle filter [11] based on Septier et al.’s state modeling [10] with a birth/death process. This state modeling enables the proposed method to deal with an unknown, time-varying number of targets.

II. PROPOSED PARTICLE FILTER FOR MULTI-TARGET TRACK-BEFORE-DETECT

This section describes the proposed particle filter for multitarget track-before-detect. It is based on a state-space model with time index $t$, unknown states $\mathbf{x}_t (t = 0, 1, 2, \ldots)$, a given initial distribution $p(\mathbf{x}_0)$, a given transition distribution $p(\mathbf{x}_t | \mathbf{x}_{t-1})$, given observations $\mathbf{z}_t (t = 1, 2, \ldots)$, and a given observation distribution $p(\mathbf{z}_t | \mathbf{x}_t)$.

A. Modeling Multi-Target States

Our states and their modeling follow Septier et al. [10]. In MTT, a target may enter/leave the region observed by
Here, survival and birth, respectively, where \( p \) and \( R \) and have a given, possibly non-Gaussian distribution \( p \). We assume that the transition distribution factorizes as

\[
p(x_t, a_t | x_{t-1}, a_{t-1}) = \prod_{j=1}^{n_{\text{max}}} p(x_{jt}, a_{jt} | x_{jt-1}, a_{jt-1})
\]

\[
\prod_{j=1}^{n_{\text{max}}} P(a_{jt} | a_{jt-1}) p(x_{jt} | x_{jt-1}, a_{jt-1}, a_{jt-1}).
\]

The factor \( P(a_{jt} | a_{jt-1}) \) is modeled by a transition probability matrix

\[
\begin{pmatrix}
1 - \pi_b & \pi_b \\
\pi_d & 1 - \pi_d
\end{pmatrix},
\]

where \( \pi_b \) and \( \pi_d \) are birth and death probabilities (given), respectively. The factor \( p(x_{jt} | x_{jt-1}, a_{jt-1}, a_{jt-1}) \) is modeled as

\[
p(x_{jt} | x_{jt-1}, a_{jt-1}, a_{jt-1}) = \begin{cases} 
p_b(x_{jt}), & \text{if } (a_{jt}, a_{jt-1}) = (1, 1) \\
p_a(x_{jt}), & \text{if } (a_{jt}, a_{jt-1}) = (1, 0) \\
p_d(x_{jt}), & \text{if } a_{jt} = 0.
\end{cases}
\]

Here, \( p_s \) and \( p_b \) are given densities corresponding to target survival and birth, respectively, where \( p_s \) may be non-Gaussian and involve nonlinearity. As we will see later, \( p_b \) is actually not used in our particle filter at all, and therefore does not need to be specified.

### B. Modeling Observations

In our track-before-detect setting, observations \( z_t \in \mathbb{C}^{n_z} \) consist in raw sensor signals, where \( n_z \) denotes the number of sensor signals. Let us first consider the additive noise for simplicity, where \( z_t \) is modeled as \( z_t = \sum_{j=1}^{n_{\text{max}}} a_{jt} h(x_{jt}) + v_t = \sum_{j:a_{jt}=1} h(x_{jt}) + v_t \). Here, \( h(x_{jt}) \) is the target signal from target \( j \) with \( h: \mathbb{R}^{n_x} \rightarrow \mathbb{C}^{n_z} \) being a given, possibly nonlinear function, and \( \sum_{j=1}^{n_{\text{max}}} a_{jt} h(x_{jt}) \) the sum of the target signals from all active targets. Additive noise \( v_t \) is assumed to be independent from time step to time step and have a given, possibly non-Gaussian distribution \( p_v \). In this case, the observation distribution is given by

\[
p(z_t | x_t, a_t) = p_v \left( z_t - \sum_{j=1}^{n_{\text{max}}} a_{jt} h(x_{jt}) \right).
\]

This model corresponds to Mahler [14], and specifically to Nannuru et al. [16] in the Gaussian case.

However, here we consider a more general observation distribution, which is such that it depends on \( x_{kt} = (x_k, a_k) \) only through \( \sum_{j=1}^{n_{\text{max}}} a_{jt} h(x_{jt}) \). That is, we consider an observation distribution of form

\[
p(z_t | x_t, a_t) = p_o \left( z_t - \sum_{j=1}^{n_{\text{max}}} a_{jt} h(x_{jt}) \right),
\]

where \( p_o \) is a given distribution possibly involving nonlinearity and non-Gaussianity.

### C. Recursive Bayesian Estimation

In the Bayesian framework, we aim to obtain a posterior distribution \( p(x_t, a_t | z_{1:t}) \) of the states given all observations up to the current time \( t \), where \( 1 : t \) is a shorthand notation for \( 1, \ldots, t \). This can be done recursively by alternating prediction and update steps, which can be carried out for our hybrid discrete-continuous states \( (x_t, a_t) \) as well in a similar manner to [17], [22]–[24].

Suppose the posterior distribution \( p(x_{t-1}, a_{t-1} | z_{1:t-1}) \) at time \( t - 1 \) is available. The prediction step uses the transition distribution to obtain a prediction distribution \( p(x_t, a_t | z_{1:t-1}) \) by the Chapman-Kolmogorov equation:

\[
p(x_t, a_t | z_{1:t-1}) = \sum_{a_{t-1}} \int p(x_t, a_t | x_{t-1}, a_{t-1}) p(x_{t-1}, a_{t-1} | z_{1:t-1}) d x_{t-1}.
\]

Here, \( \sum_{a_{t-1}} \) denotes the sum over \( a_{t-1} \in \{0, 1\}^{n_{\text{max}}} \), and we define \( p(x_0, a_0 | z_{1:0}) := p(x_0, a_0) \) with similar notations defined analogously. The update step combines this prediction distribution with the observation distribution to obtain the posterior distribution \( p(x_t, a_t | z_{1:t}) \) at time \( t \) by the Bayes theorem:

\[
p(x_t, a_t | z_{1:t}) = \frac{p(z_t | x_t, a_t) p(x_t, a_t | z_{1:t-1})}{p(z_t | z_{1:t-1})}.
\]

Here, the normalizing constant in the denominator writes \( p(z_t | z_{1:t-1}) = \sum_{a_t} \int p(z_t | x_t, a_t) p(x_t, a_t | z_{1:t-1}) d x_t \).

### D. Particle Filter Implementation

The Bayesian recursion in (7) and (8) can be implemented by using the particle filter (also known as sequential Monte Carlo) [25]–[28]. It is a versatile framework applicable to the general nonlinear, non-Gaussian state-space model, where the posterior distribution \( p(x_t, a_t | z_{1:t}) \) is approximated by using \( n_p \) point masses (or “particles”) as

\[
p(x_t, a_t | z_{1:t}) \approx \sum_{k=1}^{n_p} w^k_t \delta_{x^k_t} (x_t) \delta_{a^k_t} (a_t).
\]

Here, \( \{x^k_t, a^k_t\}_{k=1}^{n_p} \) denotes particle locations, \( \{w^k_t\}_{k=1}^{n_p} \) probability masses located at the particle locations satisfying \( \sum_{k=1}^{n_p} w^k_t = 1 \), \( \delta_{x^k_t} (x_t) \) the Dirac delta function located at \( x^k_t \), and \( \delta_{a^k_t} (a_t) \) the Kronecker delta

\[
\delta_{a^k_t} (a_t) = \begin{cases} 1, & \text{if } a_t = a^k_t \\
0, & \text{otherwise.}
\end{cases}
\]
The particle filter recursively computes particles \( \{ x^k, a^k, u^k \}_{k=1}^{n_p} \) at each time \( t \), given particles \( \{ x_{t-1}^k, a_{t-1}^k, u_{t-1}^k \}_{k=1}^{n_p} \) at the previous time \( t-1 \) and observations \( z_t \).

There are several implementations of the particle filter, and here we use an auxiliary particle filter [29] (see also [28]). This implementation takes account of observations at time \( t \) when generating particle locations at time \( t \), and can be more effective than the simple sequential importance resampling (SIR) implementation. Algorithm 1 presents a pseudocode of one iteration of the proposed auxiliary particle filter for multi-target track-before-detect. Before applying Algorithm 1, we initialize the particles by \( \{ x_0^k, a_0^k \}_{k=1}^{n_p} \sim p(x_0, a_0), w_0^k = 1/n_p \, (k = 1, \ldots, n_p) \). It is assumed that \( h \) can be evaluated at any point, and so does \( p_o \) up to a normalizing constant. It is also assumed that it is possible to sample realizations from \( p_o, p_b \), and \( p(x_0, a_0) \). We perform ancestral sampling based on the factorization (2) to sample realizations from the transition distribution \( p(x_t, a_t \mid x_{t-1}, a_{t-1}) \).

**Algorithm 1** Proposed auxiliary particle filter for multi-target track-before-detect

\[
\begin{align*}
\text{Input:} & \quad \{ x_{t-1}^k, a_{t-1}^k, u_{t-1}^k \}_{k=1}^{n_p}, z_t \\
\text{Output:} & \quad \{ x_t^k, a_t^k, u_t^k \}_{k=1}^{n_p} \\
\text{1: } & \text{for } k = 1 : n_p \text{ do} \\
\text{2: } & \quad \text{for } j = 1 : n_{\text{max}} \text{ do} \\
\text{3: } & \quad \quad \text{Draw } \bar{a}_{jt}^k \sim P(a_{jt} \mid a_{jt-1}^k) \\
\text{4: } & \quad \quad \text{if } (\bar{a}_{jt}, a_{jt-1}^k) = (1, 1) \text{ then} \\
\text{5: } & \quad \quad \quad \text{Draw } \bar{x}_{jt}^k \sim p_s(x_{jt} \mid x_{jt-1}^k) \\
\text{6: } & \quad \quad \text{end if} \\
\text{7: } & \quad \quad \text{if } (\bar{a}_{jt}, a_{jt-1}^k) = (1, 0) \text{ then} \\
\text{8: } & \quad \quad \quad \text{Draw } \bar{x}_{jt}^k \sim p_h(x_{jt}) \\
\text{9: } & \quad \quad \text{end if} \\
\text{10: } & \quad \text{end for} \\
\text{11: } & \quad \bar{w}_t^k := \frac{p_o(z_t \mid \bar{x}^k_{jt})}{p_o(z_t \mid \bar{x}^k_{jt})} \\
\text{12: } & \quad \text{end for} \\
\text{13: } & \text{Resample from } \{ \bar{w}_t^k \}_{k=1}^{n_p} \text{ to get } \{ k^l \}_{l=1}^{n_p}, \text{ where } k^l \text{ denotes} \\
\text{the index of the parent of the } l \text{th resampled particle} \\
\text{14: } & \text{for } l = 1 : n_p \text{ do} \\
\text{15: } & \quad \text{for } j = 1 : n_{\text{max}} \text{ do} \\
\text{16: } & \quad \quad \text{Draw } a_{jt}^l \sim P(a_{jt} \mid a_{jt-1}^k) \\
\text{17: } & \quad \quad \text{if } (a_{jt}^l, a_{jt-1}^k) = (1, 1) \text{ then} \\
\text{18: } & \quad \quad \quad \text{Draw } x_{jt}^l \sim p_s(x_{jt} \mid x_{jt-1}^k) \\
\text{19: } & \quad \quad \text{end if} \\
\text{20: } & \quad \quad \text{if } (a_{jt}^l, a_{jt-1}^k) = (1, 0) \text{ then} \\
\text{21: } & \quad \quad \quad \text{Draw } x_{jt}^l \sim p_h(x_{jt}) \\
\text{22: } & \quad \quad \text{end if} \\
\text{23: } & \quad \text{end for} \\
\text{24: } & \quad s_t^l := \sum_j a_{jt}^l \quad h(x_{jt}^l) \\
\text{25: } & \quad w_{t-1}^l := \frac{p_o(z_t \mid s_t^l) w_t^k}{p_o(z_t \mid \bar{x}^k_{jt})} \\
\text{26: } & \quad \text{end for} \\
\text{27: } & \text{Normalize } \{ w_{t-1}^l \}_{l=1}^{n_p} \text{ so that } \sum_{l=1}^{n_p} w_{t-1}^l = 1 \\
\end{align*}
\]

The particle filter recursively computes particles \( \{ x^k, a^k, u^k \}_{k=1}^{n_p} \) at each time \( t \), given particles \( \{ x_{t-1}^k, a_{t-1}^k, u_{t-1}^k \}_{k=1}^{n_p} \) at the previous time \( t-1 \) and observations \( z_t \).

E. Point Estimation

Once a particle representation of the posterior probability is obtained, it can be used to compute various point estimates of the states. In this paper, we focus on minimum mean square error (MMSE) type estimates. The MMSE estimate of \( x_{jt} \) conditional to \( a_{jt} = 1 \) can be computed by

\[
\hat{x}_{jt}^{\text{MMSE}} := \mathbb{E}[x_{jt} \mid z_{1:t}, a_{jt} = 1] = \frac{\sum_{k=1}^{n_p} w_{jt}^k a_{jt} x_{jt}^k}{\sum_{k=1}^{n_p} w_{jt}^k a_{jt}^k}. \tag{11}
\]

Moreover, the MMSE estimate of \( a_{jt} \) can be computed as

\[
\hat{a}_{jt}^{\text{MMSE}} := u \left( \frac{\sum_{k=1}^{n_p} w_{jt}^k a_{jt}^k - 1}{2} \right), \tag{12}
\]

where \( u \) denotes the step function

\[
u(x) := \begin{cases} 1, & x \geq 0 \\ 0, & x < 0. \end{cases} \tag{13}
\]

III. SIMULATION: MTT FOR RADIO-FREQUENCY TOMOGRAPHY

As an example, we considered a challenging task of MTT for an unknown, time-varying number of targets with unknown initial positions in the context of radio-frequency (RF) tomography as in [16]. RF tomography [30], [31] aims to localize targets (e.g., persons) in a surveillance region by using a network of RF antennas. As in Fig. 1, we employed \( n_a = 24 \) antennas (nodes) on the perimeter of a square surveillance region of dimensions \( 20 \text{ m} \times 20 \text{ m} \). In RF tomography, signals communicated between RF antennas are used instead of those emanating from targets. These signals contain target location information in the form of attenuation in received signal strength (RSS), which can be exploited for localization/tracking. Hence, we used as sensor signals \( z_t \), RSS
attenuation for all \( n_z = \frac{1}{2} n_a (n_a - 1) = 276 \) antenna pairs (links). On the other hand, states \( \mathbf{x}_t \) consisted of kinematic states \( \mathbf{x}_{jt} = (x_{jt}, y_{jt}, \dot{x}_{jt}, \dot{y}_{jt}) \) and an activity state \( a_{jt} \) of each target \( j \), where \( x_{jt} \) and \( y_{jt} \) are Cartesian coordinates of target \( j \) and \( \dot{x}_{jt} \) and \( \dot{y}_{jt} \) its velocities.

Transition of the states \( \mathbf{x}_{jt} \) for a surviving target was described by a linear Gaussian model \( \mathbf{x}_{jt} = \mathbf{F} \mathbf{x}_{jt-1} + \mathbf{G} \mathbf{w}_{jt-1} \) [16]. Here,

\[
\mathbf{F} := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{G} := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} T^2 \\ T \end{pmatrix},
\]

\( \otimes \) is the Kronecker product, \( T = 0.25 \) s the sampling period, and \( \mathbf{w}_{jt-1} \) zero-mean white Gaussian noise with covariance matrix \( \Sigma_w = 0.35 \mathbf{I} \). The distribution \( p_b \) for a newborn target was modeled by \( p_b(x, \dot{x}, \dot{y}) = \mathcal{U}(x, \dot{x} \mid 0, 1) \mathcal{N}(\dot{y} \mid 0, 1) \), where \( \mathcal{U} \) denotes the uniform distribution over the surveillance region. The transition probability matrix (3) for \( a_t \) was given by \( \pi_b = 0.2 \) and \( \pi_d = 0.1 \). The initial state distribution was modeled by \( p_0(x_0, a_0) = \prod_{j=1}^{n_a} \{ p(x_{j0}) P(a_{j0}) \} \), where \( p(x_{j0}) \) was defined in the same way as \( p_b \) and \( P(a_{j0} = 0) = 1 \). The observation distribution was given by the superpositional model in (5) with zero-mean white Gaussian noise with covariance matrix \( \Sigma_v = \sigma^2_v \mathbf{I} \). The nonlinear function \( h = (h_1, \ldots, h_{n_v}) \) was given by \( h_i(\mathbf{x}) = 5 \exp\left(\frac{-d_i(\mathbf{x})}{\sigma^2_v}\right) \), where \( d_i(\mathbf{x}) \) is an elliptical distance [32] between the \( i \)th link and a target with states \( \mathbf{x} \) and \( \phi = 5 \) and \( \sigma_v = 0.2 \) are empirically determined hyperparameters.

Sensor signals were generated as follows. Temporal behavior of \( a_t \) was deterministically scheduled so that the number of active targets started from one, then increased gradually up to four, and finally decreased gradually down to one. On the other hand, \( \mathbf{x}_t \) was randomly generated based on the above model, and so was \( \mathbf{z}_t \). We adjusted \( \sigma^2_v \) to give a desired signal-to-noise ratio (SNR), where \( \text{SNR}(\text{dB}) := 10 \log_{10}\langle \sum_{j=1}^{n_{max}} a_{jt} h(x_{jt}) \rangle^2 - 10 \log_{10}\langle \|v_j\|^2 \rangle \) with \( \langle \cdot \rangle \) being temporal averaging. The number of time steps were 200, corresponding to 50 s. Figure 2 shows an example of observed signals.

The proposed method was compared with an approximate \( \Sigma \)-CPHD filter [16], which we hereafter call the conventional method. The number of particles was fixed to \( n_p = 2000 \) in the proposed method, and time-varying with 500 particles per target plus 500 particles for proposing newborn targets in the conventional method. In both methods, an auxiliary particle filter with residual resampling [33] was employed. The maximum possible number of targets was set to \( n_{max} = 4 \) in the proposed method, and to 10 in the conventional method. In the conventional method, the probability of a target being born was set to 0.03, and the probability of each target surviving to 0.985.

Figures 3 and 4 show estimated x- and y-coordinates versus time for the proposed and the conventional methods, respectively. These estimates were obtained by MMSE estimation and k-means clustering in the proposed and the conventional methods, respectively. Particles are also shown by gray dots with associated weights expressed by darkness. Figure 5 shows the estimation error in terms of optimal subpattern assignment (OSPA) metric [34] as a function of the SNR. The error bar shows (the mean) ± (one standard deviation) for 100 trials.

IV. Conclusion

In this paper, we proposed a particle filter for multitarget track-before-detect using superpositional sensor signals. A simulation example of MTT for RF tomography clearly showed effectiveness of the proposed method. Future work includes state augmentation with unknown signal amplitudes [20], [21] and estimation of static parameters (e.g., \( \pi_b \) and \( \pi_d \)).