07 What happens if every player rushes selfishly?

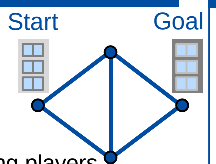
Equilibrium computation of congestion games

Abstract

In a road or telecommunication network, an edge (link) may be congested and may incur more cost if many people use it. We can compute how much each edge will be congested if each user of such network chooses a path (route) selfishly, i.e. chooses a path with minimum cost without any guidance or control. The computation of congestion requires calculation of probability for each of all available paths, which is generally prohibitive because there are a great many number of paths. To overcome this problem, we use a data structure called binary decision diagram to represent all available paths compactly. With this data structure, we can speed up the computation dramatically, and can compute such state of a network with realistic size. This study enables us to predict the state of congestion for a road and telecommunication network in a simple way, which is useful for designing such networks.

Congestion Games and Equilibrium State

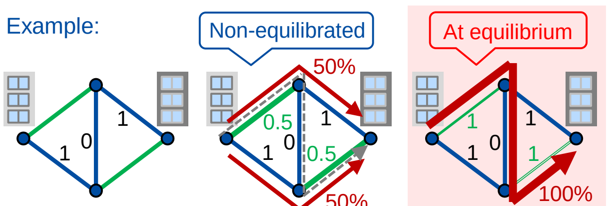
- There are infinite num. of players.
- Each player chooses a path.
  - Chooses a path with lower cost.
- Available paths are fixed.
- Edge cost increases w.r.t. mass of using players.



-> What path is chosen by each player?

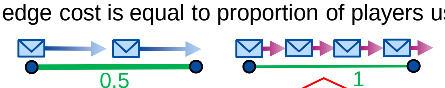
Equilibrium state : each player chooses a path with (currently) minimum cost from available paths

Example:



*Blue : edge cost is fixed

*Green : edge cost is equal to proportion of players using it

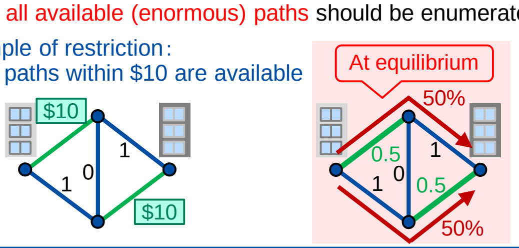


Increasing cost w.r.t. mass of using players

Computing Equilibrium

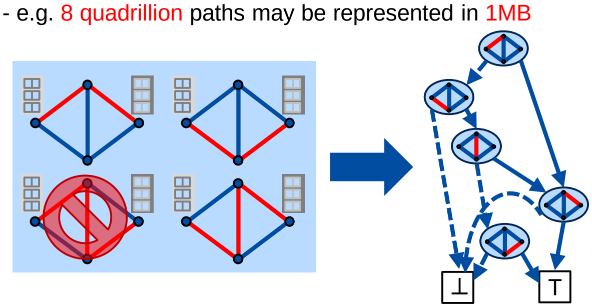
- All paths are available -> easy to compute
- Available paths are restricted -> difficult to compute since all available (enormous) paths should be enumerated

Example of restriction : Only paths within $10 are available

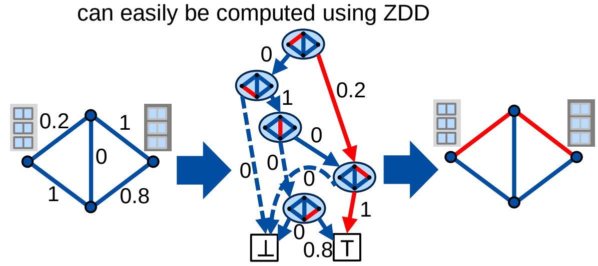


Solution with Decision Diagrams

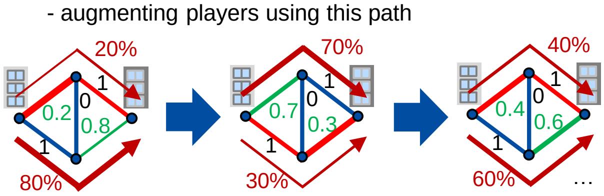
Using zero-suppressed binary decision diagram (ZDD), all available paths can be represented in compact form
- e.g. 8 quadrillion paths may be represented in 1MB



Key 1: the path with minimum cost among available ones can easily be computed using ZDD



Key 2: equilibrium can be computed by performing the following steps repetitively:
- computing path with currently minimum cost
- augmenting players using this path



References

[1] K. Nakamura, S. Sakaue, N. Yasuda, “Practical Frank—Wolfe method with decision diagrams for computing Wardrop equilibrium of combinatorial congestion games,” *Proc. 34th AAAI Conference on Artificial Intelligence（AAAI）*, 2020.

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