

# 08

## Handle a huge quantum world through a tiny window

### Investigation of the ability of indirect quantum controls

#### Abstract

It is difficult to manipulate an entire large quantum system directly. When we try to do so, huge noise will be injected into the system. However, if we can indirectly control the system via a restricted part of it, we will be able to suppress the injected noise. In this research, **we investigated the effect of the restriction mathematically**, and succeeded in completely categorizing the set of operations in the case of indirect control. This result indicates that, if the degree of the freedom of the controllable part is **more than two**, **we can universally control the whole quantum system in effect even when the degree of freedom of the uncontrollable part is very large**. This knowledge provides **a new strategy for constructing a noise-less quantum computer** or any other noise-less device for quantum information processing. If we can construct such a device, we can realize quantum information processing, e.g. factorization of huge numbers with a quantum computer.

#### Current Quantum Computer

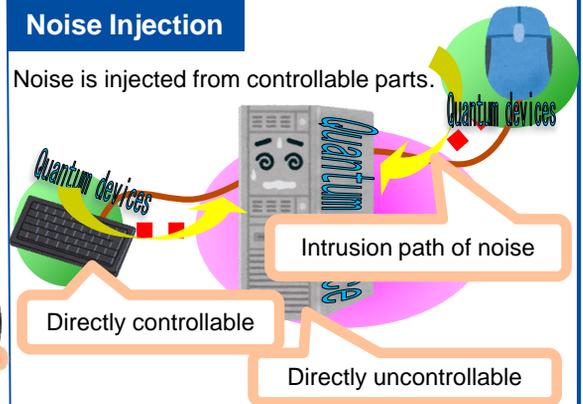


- Company I and Company G construct 53-qubit quantum computers with the noise.
- Noise increases as the number of qubits increases.
- To factorize a thousand-bit integer, millions of qubits are needed even when the noise is small.



#### Noise Injection

Noise is injected from controllable parts.



New strategy: **Prevent noise injection at the sacrifice of controllability**

#### Theoretical approach for noise reduction

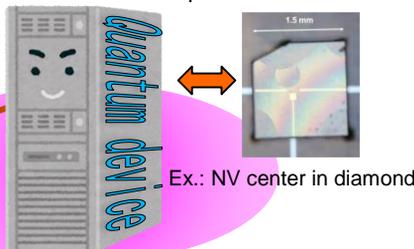
A quantum computer can be made as a composite system consisting of a directly controllable component and a directly uncontrollable component with a spontaneous interaction between them.



Discovered evidence: **When the degree of freedom of the controllable component is more than two, we can completely control the whole system in effect for any spontaneous interaction, though we can not when it is two.**

➔ **With this strategy, the noise could be suppressed even if the total number of qubits is drastically increased!**

—Question—  
Can the whole composite system be controlled even if the directly controllable component is small?



#### Mathematical basis

In the case of indirect control of a quantum system, we prove the fact that a set of all the executable operations (matrices) for a fixed interaction can be written as  $L \approx \mathcal{L}(su(d_s) \otimes J \cup i I \otimes [J, J])$  where the set  $J$  must satisfy a certain condition.

The condition depends on the degree of freedom of the controllable component.

#### References

- [1] G. Kato, M. Owari, K. Maruyama, "Algebra and Hilbert space structures induced by quantum probes," *Annals of Physics*, 412 (2020) 168046
- [2] G. Kato, M. Owari, K. Maruyama, "Hilbert space structure induced by quantum probes," *Proceedings of the 11th Italian Quantum Information Science conference (IQIS2018)*, Catania, Italy, 12 (2019) 4

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