

BLIND SEPARATION OF REFLECTIONS USING SPARSE ICA

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ABSTRACT

We address the problem of Blind Source Separation (BSS) of superimposed images and, in particular, consider the recovery of a scene recorded through a semireflective medium (e.g. glass windshield) from its mixture with a virtual reflected image. We extend the Sparse ICA (SPICA) approach to BSS and apply it to the separation of the desired image from the superimposed images, without having any *a priori* knowledge about its structure and/or statistics. Advances in the SPICA approach are discussed. Simulations and experimental results illustrate the efficiency of the proposed approach, and of its specific implementation in a simple algorithm of a low computational cost. The approach and the algorithm are generic in that they can be adapted and applied to a wide range of BSS problems involving one-dimensional signals or images.

1. INTRODUCTION

The phenomenon of a virtual image, being semireflected by a transparent medium, situated along the optical axis somewhere between the imaged scene and the observing point, and superimposed on the imaged scene, is typical of many optical setups. It may arise, for example, when photographing objects behind a glass window or windshield, since most types of glass have semireflecting properties. The need to separate the contributions of the real and the virtual images to the combined, superimposed, images has importance in applications where reflections may create ambiguity in scene analysis. Separation of the desired image from the superimposed reflection is of particular importance in systems implementing vision algorithms based on feature matching. Also, since the virtual and the real images may appear at different distances from the camera, the reflections may confuse autofocusing devices [12].

Approaches to reconstruction of the virtual and the real images, based on polarimetric imaging, have attracted attention during the last few years [4], [11]. Incorporation of a polarizer into the optical system is a common photographic technique allowing suppression of semireflective layers [12]. Several constructions of such cameras, e. g. a system equipped with a liquid crystal polarizer [7], were recently proposed.

However, the polarizer is capable of removing the reflected component completely only when the viewing angle is equal to the Brewster's angle. This case results, however, in severe geometric distortions. In other cases, the polarization is not sufficient [5], [6], [12]; even when the polarizer is oriented to minimize the reflected component, the virtual image is still visible.

Several signal post-processing approaches were proposed in recent studies, however, they rely mainly on motion, stereo and

focus, and assume that the real and the virtual objects lie at significantly different distances from the camera [3], [13]. Other methods assume some knowledge about the scene, such as the refraction index, which makes them hardly feasible in the general case [12].

Farid and Adelson [5], [6] proposed to use an analytic version of independent component analysis (ICA) for blindly separating the reflected and the transmitted images. Such an approach does not require any prior knowledge regarding model parameters, and offers better feasibility in real-world applications. However, the proposed method is not general enough, since it works with two sources only. On the other hand, iterative approaches such as the information maximization (Infomax) algorithm [2] are relatively slow, although they can handle any number of sources, provided a sufficient number of mixtures are available.

It has been recently demonstrated by Zibulevsky et al [14], [15], [8] that the assumption of sparseness is very powerful and can significantly improve the accuracy and the computational efficiency of existing ICA algorithms. In addition, sparse decomposition allows using simple "geometric" algorithms to separate the mixed data. We adopt the Sparse ICA (SPICA) approach and show that it affords effective separation of a transmitted image from superimposed reflections [1].

We first formulate in Section 2 the problem of semireflections as a linear blind source separation problem, and then proceed to discuss in Section 3 the motivation for utilizing sparse representations for the purpose of blind source separation. This leads to the presentation of the SPICA method. Section 4 is dedicated to the question of how to obtain a sparse decomposition of images in order to exploit the advantages of the sparseness. We introduce several new approaches, suitable for decomposition of natural images. In Section 5, we apply the algorithm to simulated and real-world images and compare it with the previously proposed approaches.

2. THE BLIND SOURCE SEPARATION PROBLEM

In a typical BSS task, N mixtures are observed or received. Each of these available signals (or images), is assumed to be generated by a linear mixture of unknown *sources*, where the number of sources is usually assumed to be known to the observer, but this condition can be relaxed. Thus, an N -dimensional vector of observed signals is generated by the product of an unknown $N \times M$ *mixing matrix* A and an M -dimensional vector of unknown source signals. The task is to estimate the mixing matrix and then recover the source signals.

A typical embodiment of this problem in the context of superimposed reflections is depicted in Figure 1. The real object (a) is situated on the optical axis behind a semireflecting planar

lens (d), inclined with respect to the optical axis [12]. Another object (b) is partially reflected by the lens, creating a virtual image (c). The camera (f) records a superposition of the two images. Thus, the intensity of the observed mixed image is given by:

$$m_1 = a_{11}s_1 + a_{12}s_2 \quad (1)$$

where s_1 and s_2 are the images of two source objects (a) and (b), and a_{11} , a_{12} are constants, and their specific values depend on the optical geometry and properties of the reflective medium. It is assumed here that the problem is spatial invariant. This reasonably good approximation of the physical conditions can be relaxed.

Note that in the setup described so far we have a 1×2 mixing matrix. Since the reflected light is polarized, by introducing a linear polarizer (e), the relative weights of the two mixed images can be altered, thus yielding mixtures of the form

$$m_n = a_{n1}s_1 + a_{n2}s_2 \quad : \quad n = 1, \dots, N \quad (2)$$

or in matrix notation:

$$M = A \cdot S \quad (3)$$

where m_1, \dots, m_n are the N mixed images and s_1 and s_2 are the source images represented as row vectors and A is the mixing (crosstalk) matrix.

The mixing matrix is usually unknown, unless side information regarding the physics of the mixing medium is available. In the context of the mixed reflections problem, this would require the availability of an exact optical model of the imaging system, visual scene and the medium. We, of course, do not assume any prior knowledge regarding the mixing matrix. Our goal is to determine the two source images S from the set of equations (3) with an unknown mixing matrix. This is the essence of the BSS problem.

Under the assumption that the sources are statistically independent (which is reasonable in the presented case), it is possible to recover sources s_1 and s_2 up to a permutation and multiplicative constant, by estimating the mixing matrix $\tilde{A} \approx A$, and estimating the sources by its inversion:

$$\tilde{S} = \tilde{A}^{-1} \cdot M \quad (4)$$

This problem is solved using the Sparse ICA method, discussed in the following section.

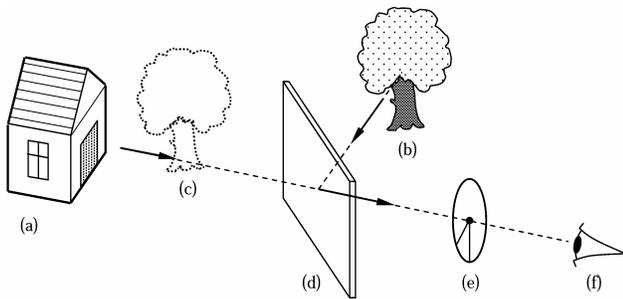


Figure 1 – A typical optical setup including a semireflector: (a) – object 1, (b) – object 2, (c) – virtual object, (d) – glass, (e) – polarizer, (f) – camera.

3. SPARSE ICA (SPICA)

Zibulevsky et al [14] have noticed that in case of *sparse* sources, their linear mixtures can be easily separated using very simple

“geometric” algorithms. Given linear mixtures resulting from source images with the majority of pixels having a near-zero magnitude and under the assumption of statistical independence of the locations of the non-zero pixels in the sources, there is a high probability that only a single source will contribute to a given pixel in each mixture. Consequently, the majority of the pixels in each mixture will be influenced by one source only and have a magnitude equal to that of the source multiplied by the corresponding coefficient of the mixing matrix. In the scatter plot of one mixture versus the other these pixels will therefore lie along a line (each corresponding to a source) at a distance from the center depending on source magnitude (see Figure 2). Hence, it is possible to reveal the ratios of each source’s contribution to the mixtures by measuring the angles of each of the lines [1].

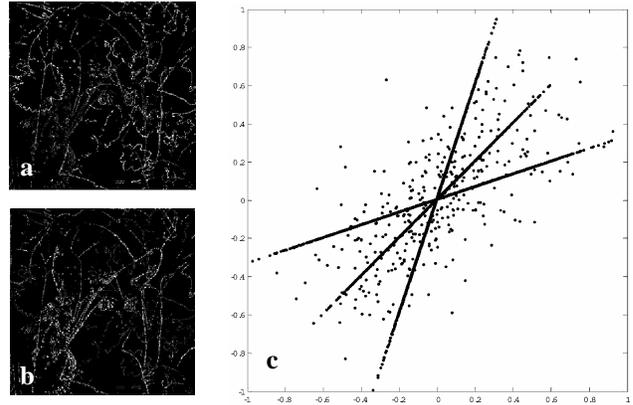


Figure 2 – (a)–(b) Sparse sources, (c) Scatter plot of the mixtures m_2 vs m_1 .

Geometric separation approaches are based on the detection of co-linearities in the distribution of the coefficients over the scatter plot. The straightforward way to recover the proper orientations from the scatter plot is by using the angular histogram. Applications of this approach are limited to low dimensions (practically, to 2D) due to the difficulty to construct the angular histogram in higher dimensions. The M points in the scatter plot (M equals the number of pixels in the image) are represented as points in N -dimensional space (in our case, $N = 2$). Since the mixtures are assumed to have a zero mean, the oriented co-linear distributions are centered at the origin. For each point $c_k \in \mathbb{R}^2$, the angle

$$\alpha_k = \tan^{-1}(c_k^2/c_k^1) \quad (5)$$

is computed. Building the histogram of α , it is possible to detect the directions using a peak-detection algorithm.

An alternative approach is clustering along orientations of data concentration in the scatter plot. Each point c_k is projected on a unit hemisphere, by normalizing the data vectors:

$$c_k = \frac{c_k}{\|c_k\|} \quad (6)$$

and multiplying them by the sign of the first vector coordinate c_k^1 [8]. As the result, a number of clusters corresponding to the number of the sources is formed on the hemisphere. Applying some clustering algorithm, e.g. Fuzzy C-Means (FCM), it is possible to determine the cluster centers. The coordinates of the

centers define the columns of the estimated mixing matrix, equivalent to the orientations found in the previous approach. Refinement iterations may be performed by deleting the points with low membership value and performing again FCM on the refined data set.

Lennon et al [9] used a dynamic vector quantization algorithm instead of clustering. This algorithm may be advantageous over FCM, since it does not assume an *a priori* knowledge of the number of sources.

It is important to note that the performance of geometric separation depends heavily on the quality of the scatter plot; The better the orientations in the scatter plot are distinguishable and clear, the higher is the accuracy in estimating the mixing matrix.

4. SPARSE DECOMPOSITION OF IMAGES

In Section 3 we showed simple geometric algorithms capable of separating mixtures of sparse images. It is obvious, however, that the sources in most applications, including the semireflective layer separation problem are natural images and have rather non-sparse nature. However, such images can be sparsely represented, i.e. there exists a linear transformation T such that

$$d_i = T s_i \quad (7)$$

is sparse. (Note that there is not necessary to be able to restore s_i from d_i , that is T does not necessarily have to be invertible). Application of the transformation to the mixtures in (3), due to the linearity of T , yields

$$T m_i = T(a_{i1} s_1 + a_{i2} s_2) = a_{i1} d_1 + a_{i2} d_2 \quad (8)$$

Thus, the problem at hand is equivalent to separation of linearly-mixed sparse sources. This can be solved using the techniques described in Section 3.

The question is then whether there exists a universal transformation that projects any given natural signal or image into its optimal sparse representation. Unfortunately, this is not the case. Different classes of signals require their specific optimal [8] sparse transformations. However, a wide range of transforms can result in sparse representation that permits good estimation of the mixing matrix. For example, and this is relevant to our reflection separation problem, in the case of natural images (Figure 4a-b), the edge distribution is usually sparse (other than in the extreme cases of densely textured image).

Thus, even such a simple operation like a derivative will yield a sparse image (Figure 4e-f). The effect of this simple transformation on the potential of separation can be appreciated by considering the scatter plots of the original mixtures (Figure 5a) and comparing it with the scatter plot of the transformed mixtures (Figure 5b). The two dominant orientations, corresponding to the columns of the mixing matrix, are apparent in the scatter plot of the transformed mixtures. To compare with, the cloud in the scatter plot of the original images does not permit any reasonable estimation of the mixing matrix. The reason is that original images are endowed with pixel values (different from zero) distribution all over the image space, and therefore the probability of having a high value at a specific pixel in one image, and almost zero value in the corresponding pixel of the other image is very small.

On the other hand, since it is very unlikely that the majority of the edges of the source images will coincide, many points in

the scatter plots of the mixtures are contributed by a single source only. Thus, optimality of the sparse transformation is the search for such a transformation that increases the proportion of those points contributed by only either one of the sources.

4.1. Multinode decomposition

Since there is no common sparse representation to different images, such a simple transformation as the derivative is usually data-dependent. Having this problem in mind, richer representations, that over a wide range of natural images lend themselves to relatively good sparse representations, such as the wavelet packet transform (WPT), were proposed [8].

The 2D WP image decomposition can be represented as the tree shown in Figure 4. The nodes of the WP tree are numbered by two indices (i, j) , where $i = 1, \dots, N$ is the depth of the level, and $i = 0, \dots, 2^j - 1$ in the node number at the specified level [10].

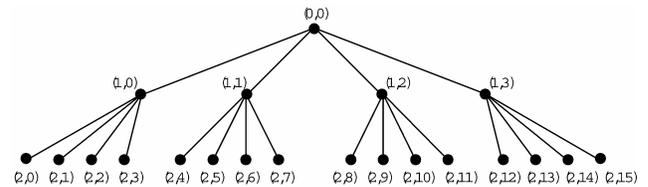


Figure 3 – 2D wavelet packet tree.

The task is to select only the nodes of the WPT, in which the decomposition is sparse. Kisilev et al [8] proposed an algorithm, in which the clustering procedure is first applied to each one of the nodes, but only nodes with minimal *global distortion* (i.e. the mean-squared distance of data points to the centers of the closest clusters) are then selected for further processing. A more general approach is to assign some quality factor to each node, which determines its sparseness and then select a certain percentage of the “best” nodes in the sense of the assigned quality criterion. The choice of such a criterion will be discussed later in Section 4.2.

As an alternative to the WP decomposition, we propose to divide the image into blocks (possibly overlapping), compute some simple sparse transformation such as the first or the second order derivative (possibly concatenated) and only then to select the “best” blocks according to some sparseness criterion. Our observation is that most natural images have certain regions, in which edges and texture make such an approach efficient. Figure 6 depicts how the use of blocks can refine the sparseness and consequently the quality of the scatter plot in the previous example shown in Figure 4. The mixtures were partitioned into 16 blocks of equal size, and the same sparse transformation was applied to each block independently.

4.2. Quantitative sparseness criteria

Finding an adequate sparseness criterion is a crucial task for selecting the best nodes or blocks. The general problem of quantitative node sparseness estimation is to find such a function $q(x)$, which given a vector $x \in \mathbb{R}^n$ returns a large value if it is sparse or a small value if it is not sparse.

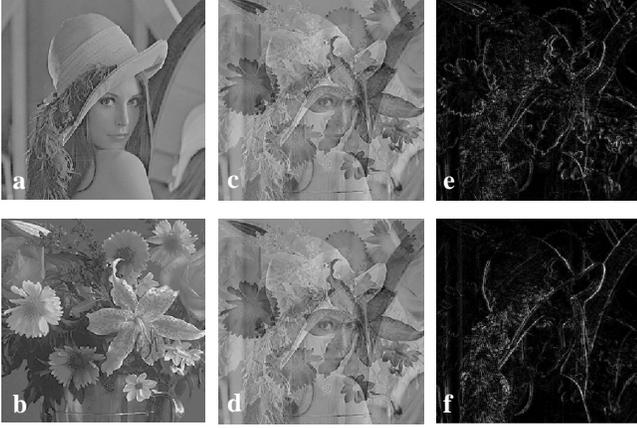


Figure 4 – (a)–(b) Non-sparse sources, (c) – (d) Synthetic mixtures, (e)–(f) The transformed mixtures obtained by the action of derivative in x-direction.

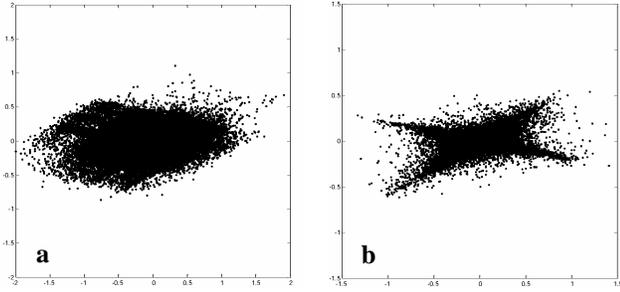


Figure 5 – Scatter plot of the mixtures m_2 vs m_1 before (a) and after (b) the sparse transformation.

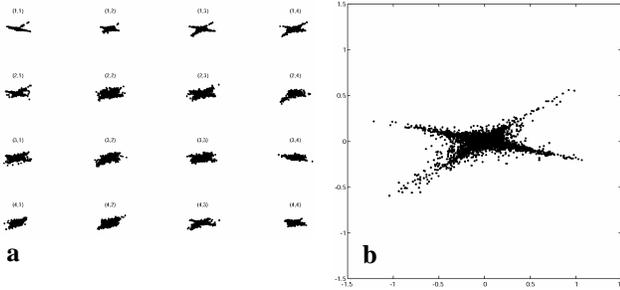


Figure 6 - Sparseness refinement by image partitioning into 16 blocks. Scatter plots of the coefficients in each block using x derivative as the sparse transformation (a) and a scatter plot resulting from merging the coefficients of blocks (1,1), (1,3) and (1,4) (b).

One of the possibilities is to use the so-called L_0 (threshold) norm, i.e. measure the number of vector coordinates, which are higher than some threshold τ :

$$q^{-1}(\mathbf{x}) = \frac{1}{n} \sum_{k=1}^n \mathbf{I}(x_k \geq \tau) \quad (9)$$

where \mathbf{I} is the indicator function. A natural choice of the threshold would be $\tau = \|\mathbf{x} - \bar{\mathbf{x}}\|_2$, where $\bar{\mathbf{x}}$ is the mean value of \mathbf{x} . Another possible sparseness criterion is the L_p norm:

$$q^{-1}(\mathbf{x}) = n^{\frac{1-p}{p}} \cdot \frac{\|\mathbf{x}\|_p}{\|\mathbf{x}\|_2} \quad : \quad 0 < p \leq 1 \quad (10)$$

Recent studies indicate that the L_1 norm is a more natural choice for dealing with various aspects of image quality criteria. This, normalized by the L_2 norm, as in (10), may turn out to be the best sparseness criterion. This, however, has yet to be further investigated.

5. RESULTS

In this section, we apply the SPICA approach to polarization images obtained by simulation and photographed in real-world conditions. For comparison, we repeated the results of Farid and Adelson [5].

5.1. Simulated data

In the first experiment, the mixtures were obtained by artificially mixing two source images (“Terry” and “Parrot”, denoting sources A and B, respectively; left column of Figure 11). We used two SPICA methods: WP decomposition and block partitioning with second-order Sobel numeric derivative applied as the projection onto the sparse representation space. The reconstruction was performed geometrically, using an angular histogram. Peak detection was performed in two stages: first, peaks locations were estimated in a coarse histogram. Then, the coarse estimation was refined by performing peak detection in a fine histogram of the data points in a small angular spread around each of the estimated peaks.

Whereas one has to scrutinize the separated images in order to realize the improvement in the results obtained by the sparse representations over blocks, proposed in this study, as compared with those obtained by the application of closed-form ICA ([5], [6]; compare the left two columns of Figure 7), the SNR values represented in Table 1 reveal a considerable improvement by sparse representations. It is difficult, however, to draw a clear conclusion from this simulation study about the comparison of the WP and the block partitioning. Obviously, all the localized techniques yield good results.

Table I – SNR (in dB) of the reconstructed sources A and B

	Farid & Adelson	SPICA (WP)	SPICA (blocks)
A	12.18	38.58	35.83
B	26.07	45.96	64.96

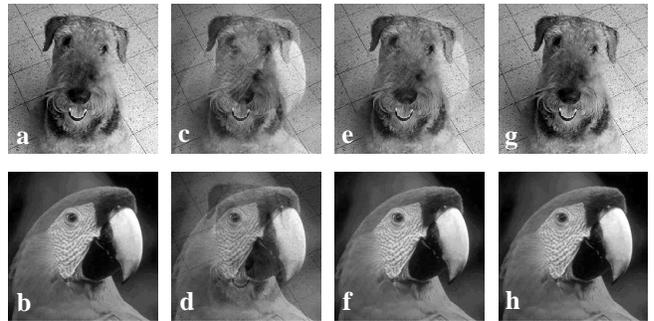


Figure 7 – Separation of synthetic mixtures. (a)-(b) Sources, (c)-(d) mixtures, (e)-(f) Reconstruction by the Farid-Adelson approach, (g)-(h) Reconstruction by SPICA with block partitioning.

5.2. Real-world data

To further test the performance of the three algorithms (the closed-form ICA and two version of SPICA), we used the images of a Renoir's painting, framed behind glass, with a superimposed reflection of a mannequin ("Sheila"), photographed by Farid and Adelson through a linear polarizer at orthogonal orientations¹. SPICA algorithms were implemented in MATLAB².

The block partitioning approach is a natural way to handle the case of spatially-varying mixing coefficients, which often occur in reality. A better assumption is of a locally spatially invariant system that can be dealt with by partitioning of the image into blocks. The acquired images were divided into four equally sized super-blocks and the separation problem was solved in each super-block separately. The coefficients of the estimated unmixing matrix were then linearly interpolated over the entire image to produce a more accurate unmixing. Figure 8 shows the reconstruction results. The desired photographed image of Renoir's painting is recovered with high precision, without notable artifacts. The non-separated details in the reconstructed image of the mannequin, resulted from geometric distortions due to imperfections of the optical system, i.e. the optical system can not be considered as a spatial-invariant system.

Consequently, the reconstruction of the virtual image is improved by block partitioning in two ways, first by better matching the algorithm to the "nonstationarity" of the sources, and second by better dealing with the spatial varying properties of the optical system. Reconstruction with WP decomposition was done under the spatial invariance assumption.

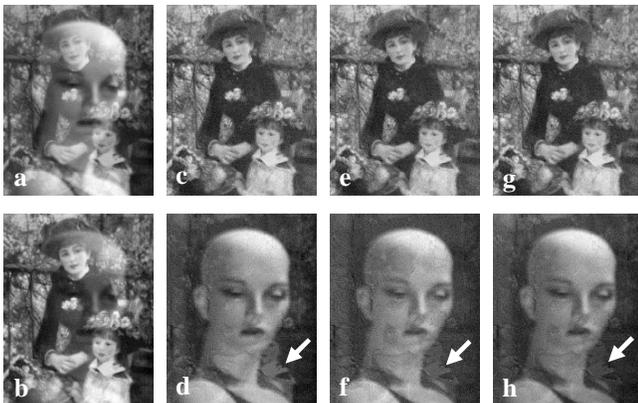


Figure 8 – Separation of synthetic mixtures. (a)-(b) Mixtures, (c)-(d) Reconstruction by the Farid-Adelson approach, (e)-(f) Reconstruction by SPICA with WP, (g)-(h) Reconstruction by SPICA with block partitioning.

Yet another important task after performing the separation of images is to determine which of them belongs to the real

object and which to the reflected one. Taking into consideration the prior knowledge of the fact that the contribution of the virtual object varies more due to polarization than the intensity of the real object (which remains almost constant), we can identify the virtual object as that corresponding to the column of the estimated mixing matrix, whose coefficients vary the most. This principle is supported by our processing of the example of the real polarized images.

6. CONCLUSIONS

The Sparse ICA approach can be effectively used in wide range of scenarios wherein various mixtures of source images are available for separation of the sources. In this study we were primarily concerned with separation of an image from virtual images superimposed on it by reflections from a semireflecting medium. The proposed novel sparse decomposition method incorporates block partitioning, suitable for nonstationary natural images, as well as for imaging systems such as polarized semireflecting media, that can not be considered as spatial invariant systems, but can to a good approximation be dealt with as locally spatial invariant systems. Experiments conducted with simulated and photographed data shows the efficiency of this approach and its advantages in the specific problem of separation of an image with superimposed reflections over previously-proposed wavelet packet decomposition and closed-form ICA.

We have assumed that only two images, acquired at perpendicular polarization angles, are available. However, in its implementation to physical systems one may extend the application to acquisition of more than two images. In this case, principal component analysis (PCA) can be used prior the application of ICA.

Lastly, the block partitioning into fully localized subsets of data can be also useful in cases where in addition to the mixing there is a weak effect of convolution. This combined case of mixing with convolution is, of course, much more difficult than the mixing only, and is encountered in many physical systems. However, convolution too is only an approximation of more complex action of a kernel in an integral equation that does not represent convolution. The argument of locally stationarity is there too valid and therefore our proposed block partitioning should improve the results whatever type of blind deconvolution is combined with the blind source separation.

7. ACKNOWLEDGEMENT

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¹ The polarized images and the analytic ICA MATLAB code for image separation are available from <http://www.cs.dartmouth.edu/~farid/research/separation.html> (courtesy of Hany Farid, Dartmouth College).

² The Sparse ICA MATLAB codes are available from <http://visl.technion.ac.il/bron/spica> (courtesy of Alexander and Michael Bronstein).

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