

# Computationally sound formal blind signature

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# Motivation

- Bridging the gap between
  - Computational, probabilistic model
  - Symbolic, non-deterministic modelof protocol security [Micciancio-Warinschi][Cortier-Warinschi]
- Symbolic model with **blind signature** [Kremer-Ryan]
  - Voting protocols and digital cash protocols

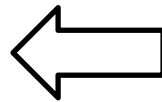
# Contributions

**1. Construct symbolic model with blind signature**

Computational model

Assumptions on  
blind signature

**2. Prove  
soundness**

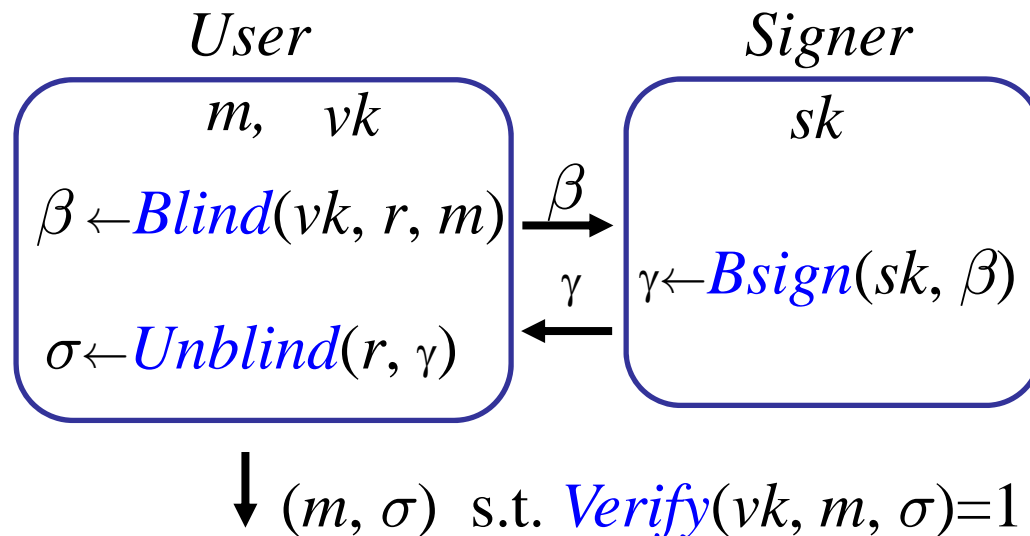


Symbolic model

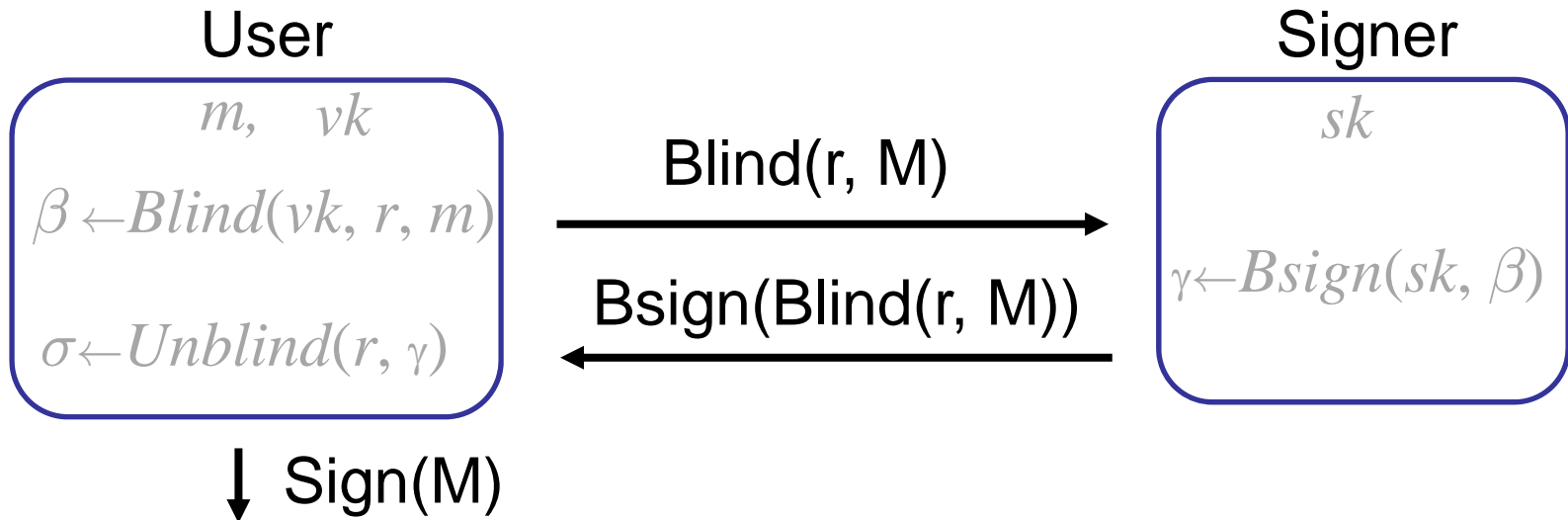
Adversary's ability

# Blind Signature Scheme

- Enables user to obtain signature  $\sigma$  to message  $m$  keeping  $m$  secret to signer (blindness)
- In voting scheme, voter is enabled to obtain ballot  $\sigma$  keeping his vote  $m$  secret to administrator



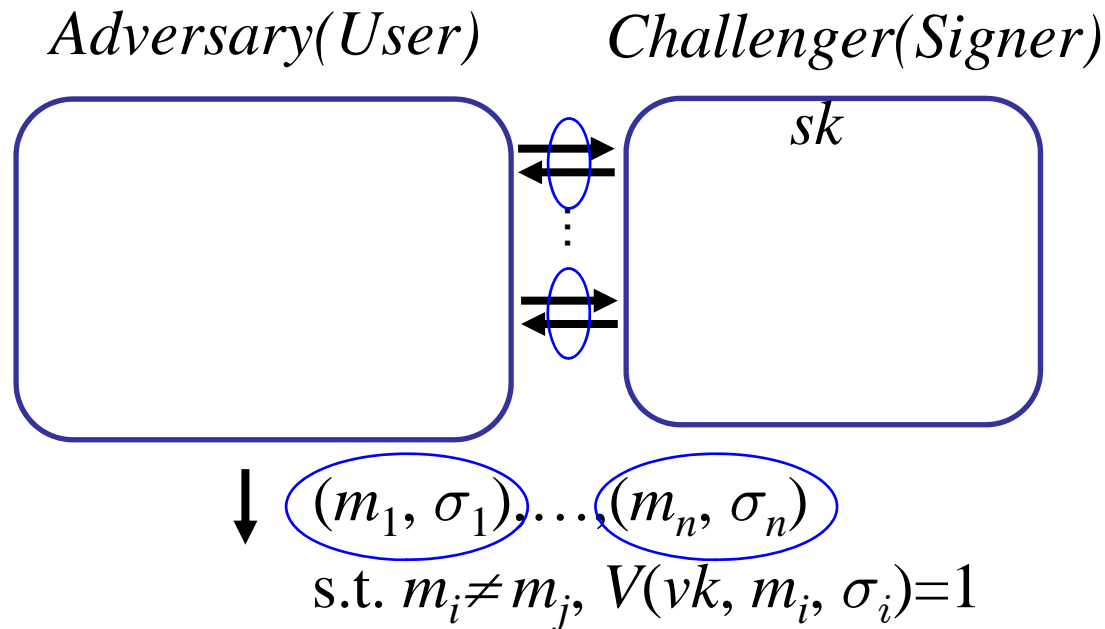
# Symbolic Blind Signature



- How can we define symbolic adversary's ability reflecting assumptions in *computational model*?

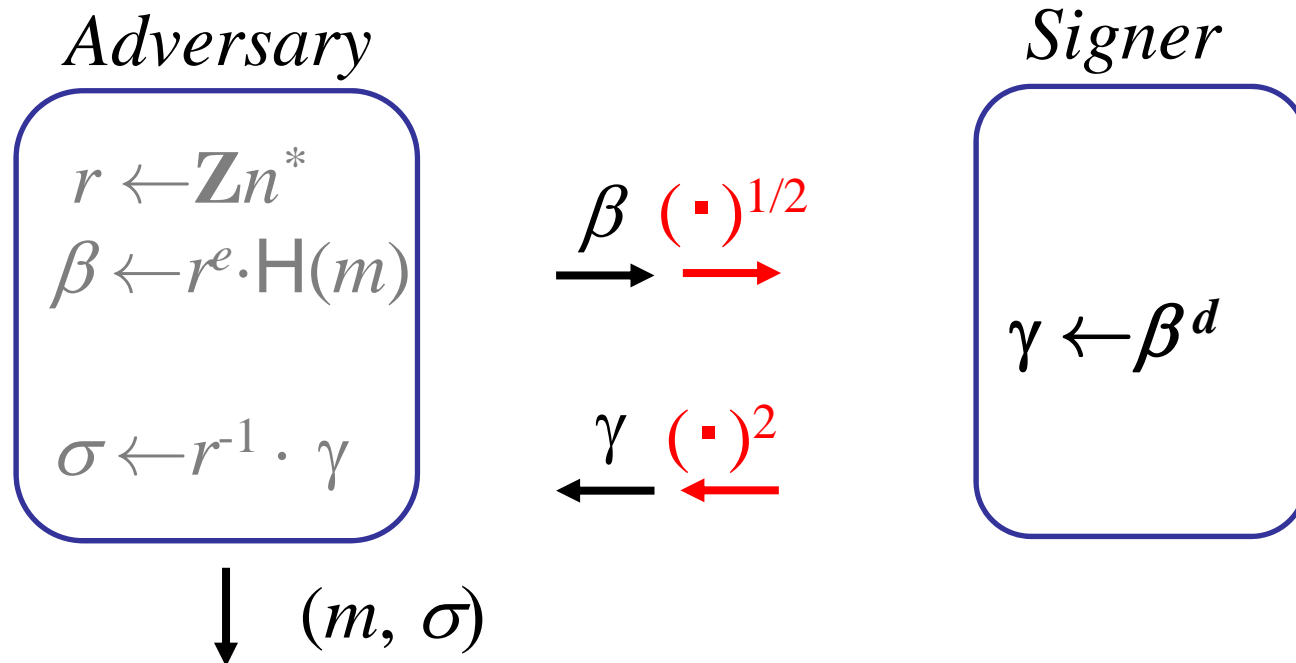
# Computational Assumptions

1. **Blindness**: Information on message  $m$  is not revealed from blinded message  $\beta \leftarrow \text{Blind}(vk, r, m)$
2. **Unforgeability**: Number of sigs. obtained by adv  $\leq$  Number of times signer signs



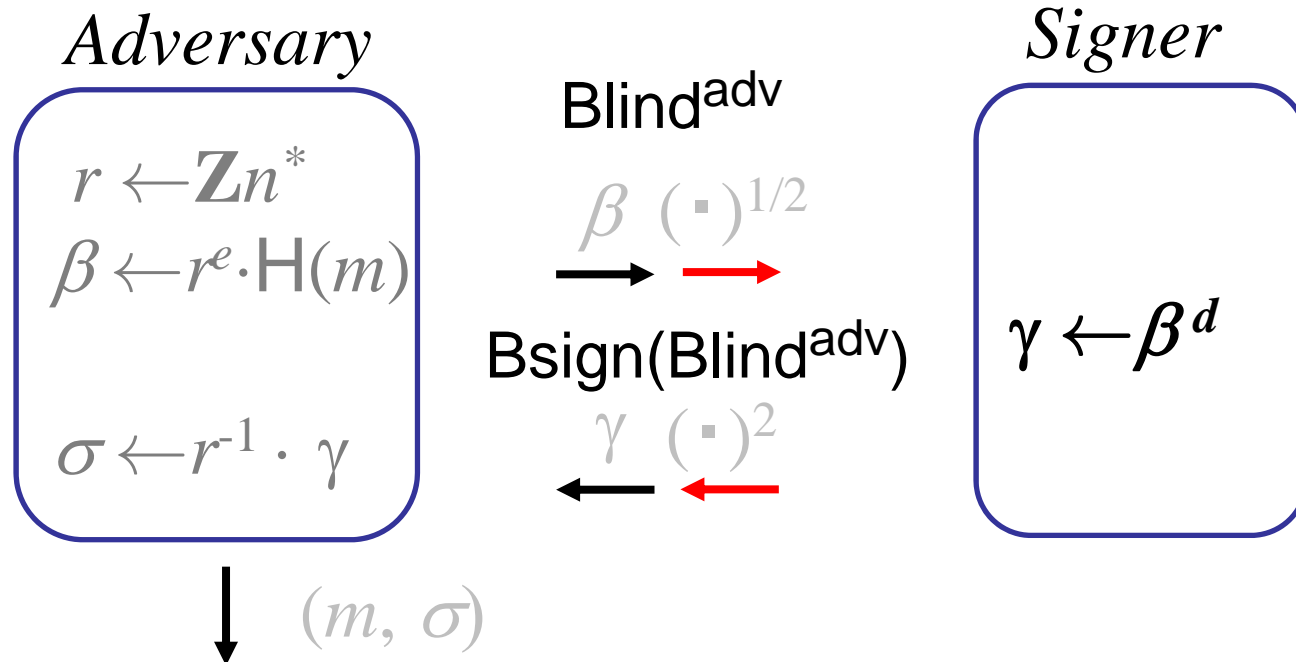
# E.g. FDH-RSA blind signature

- Adversary may not follow the scheme to obtain valid signature



# Adversary's blinded message

- Introduce adversary's blinded message  $\text{Blind}^{\text{adv}}$  to the symbolic model
- Also represents “irregular” blinded message





# Symbolic Adversary's Ability

- From a set  $\Gamma$ , adv. can obtain message  $m$ 
  1. Deducible by the rules below, where
  2. Num. of times he uses the **unblind** rules  $\leq$  Num. of  $\text{Bsign}(\text{Blind}^{\text{adv}})$  received
- No rule for  $\text{Blind}(r, M)$  because we assume honest party never disclose random  $r$

$$\frac{\Gamma \vdash C \quad \Gamma \vdash N^{\text{adv}} \quad \Gamma \vdash \text{Blind}^{\text{adv}} \quad \frac{\Gamma \vdash \text{Bsign}(\text{Blind}^{\text{adv}}) \quad \Gamma \vdash M}{\Gamma \vdash \text{Sign}^{\text{adv}}(M)}}{\Gamma \vdash \text{Sign}^{\text{adv}}(M)} \text{unblind}$$

$$\frac{M \in \Gamma}{\Gamma \vdash M} \quad \frac{\Gamma \vdash M_0 \quad \Gamma \vdash M_1}{\Gamma \vdash \{M_0, M_1\}} \quad \frac{\Gamma \vdash \{M_0, M_1\}}{\Gamma \vdash M_i} \quad \frac{\Gamma \vdash \text{Sign}(M)}{\Gamma \vdash M}$$

# Examples

- Symbolic adversary can not deduce

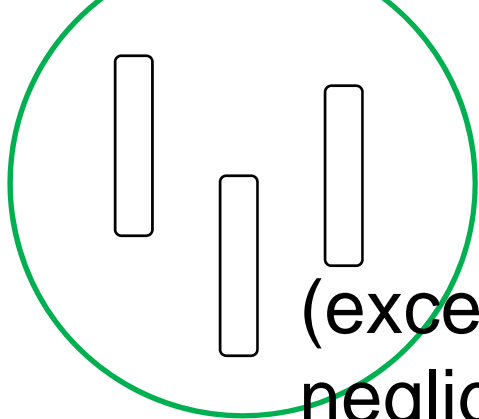
Blindness  $\left\{ \begin{array}{l} \text{Blind}(r, N) \not\vdash N \\ \text{Bsign}(\text{Blind}(r, N)) \not\vdash N \end{array} \right.$

Unforgeability

$\left\{ \begin{array}{l} M \not\vdash \text{Sign}(M) \\ \text{Bsign}(\text{Blind}^{\text{adv}}), N, N' \not\vdash \{\text{Sign}^{\text{adv}}(N), \text{Sign}^{\text{adv}}(N')\} \end{array} \right.$

# Soundness

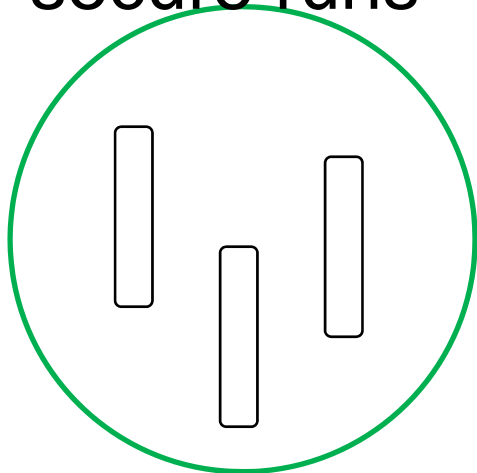
computational  
protocol runs



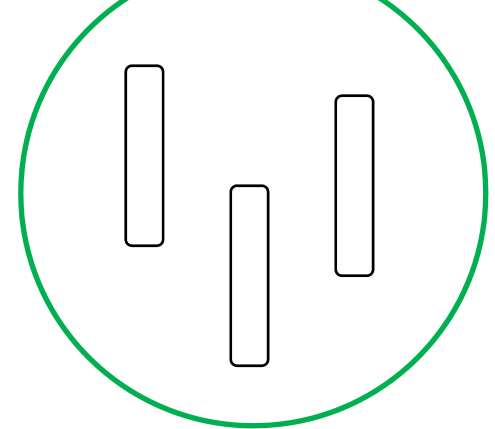
(except with  
negligible prob.)

$\supseteq$

secure runs

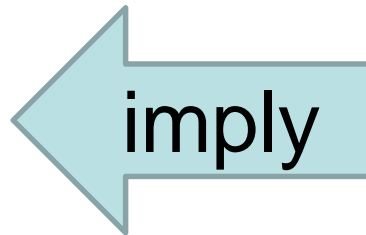
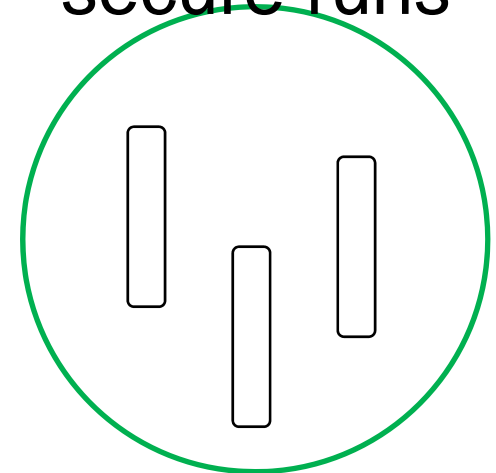


symbolic  
protocol runs



$\supseteq$

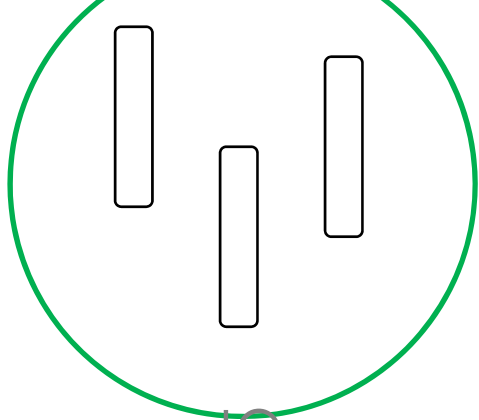
secure runs



$\sim$

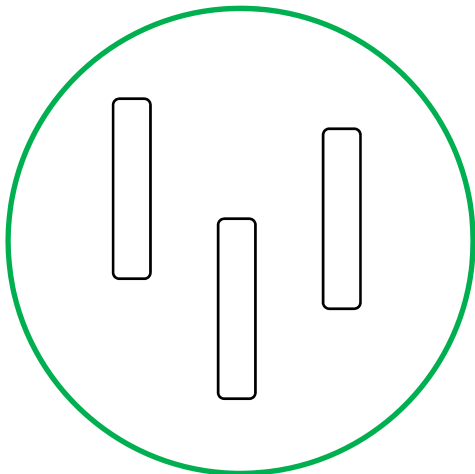
# Mapping Lemma

computational  
protocol runs

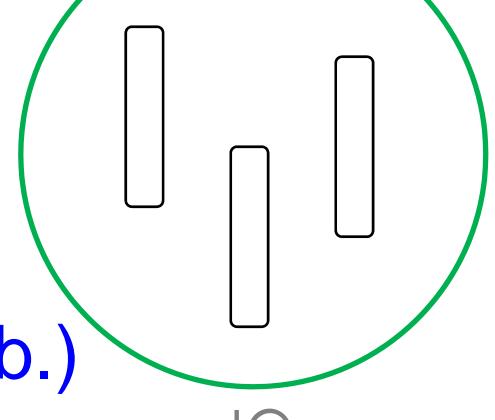


in

secure runs

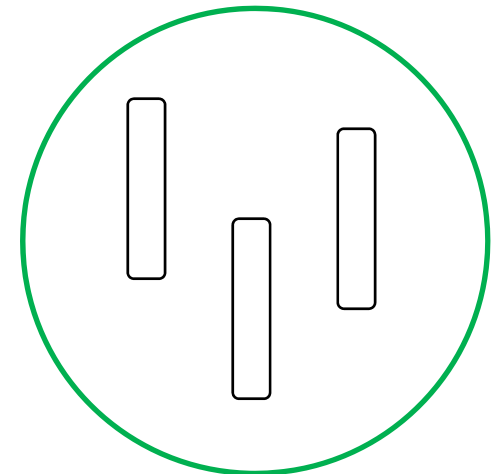


symbolic  
protocol runs



in

secure runs



$\exists f$

$\rightarrow$

$\subseteq$

(except with  
negligible prob.)

$f^{-1}$

$\leftarrow$

$\supseteq$

$\sim$

# Outline of Proof

Similar to [Cortier-Warinschi]

I. Construct mapping from computational runs into symbolic runs:

$\langle \text{“blind”}, m \rangle \mapsto \text{Blind}(r, M)$ , if generated by honest party  
 $\mapsto \text{Blind}^{\text{adv}}$ , otherwise

$\langle \text{“bsign”}, m \rangle \mapsto \text{Bsign}(\text{Blind}(r, M))$ ,

if successfully unblinded and verified

II. Show the symbolic runs satisfy the conditions

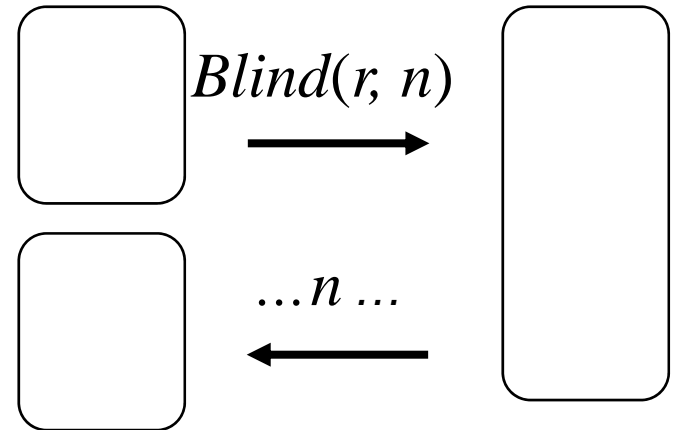
1. Message sent by adv. can be deduced by the rules

2. Num. of  $\text{sign}^{\text{adv}}(-) \leq \text{Num. of bsign}(\text{blind}^{\text{adv}})$

# If the cond.1 is not satisfied

*Honest parties*

*Adv.*



# If the cond.1 is not satisfied

*Adversary of blindness game*

*Challenger*

Choose  $b \leftarrow \{0, 1\}$

Play

$User_0(vk, n_b)$

$User_1(vk, n_{1-b})$

$n_0, n_1$



$Blind(r, n_b)$



$Blind(r, n_{1-b})$



Choose  $n_0, n_1$

*Simulate honest parties  
by using challenger*

*Adv.  
of Protocol*

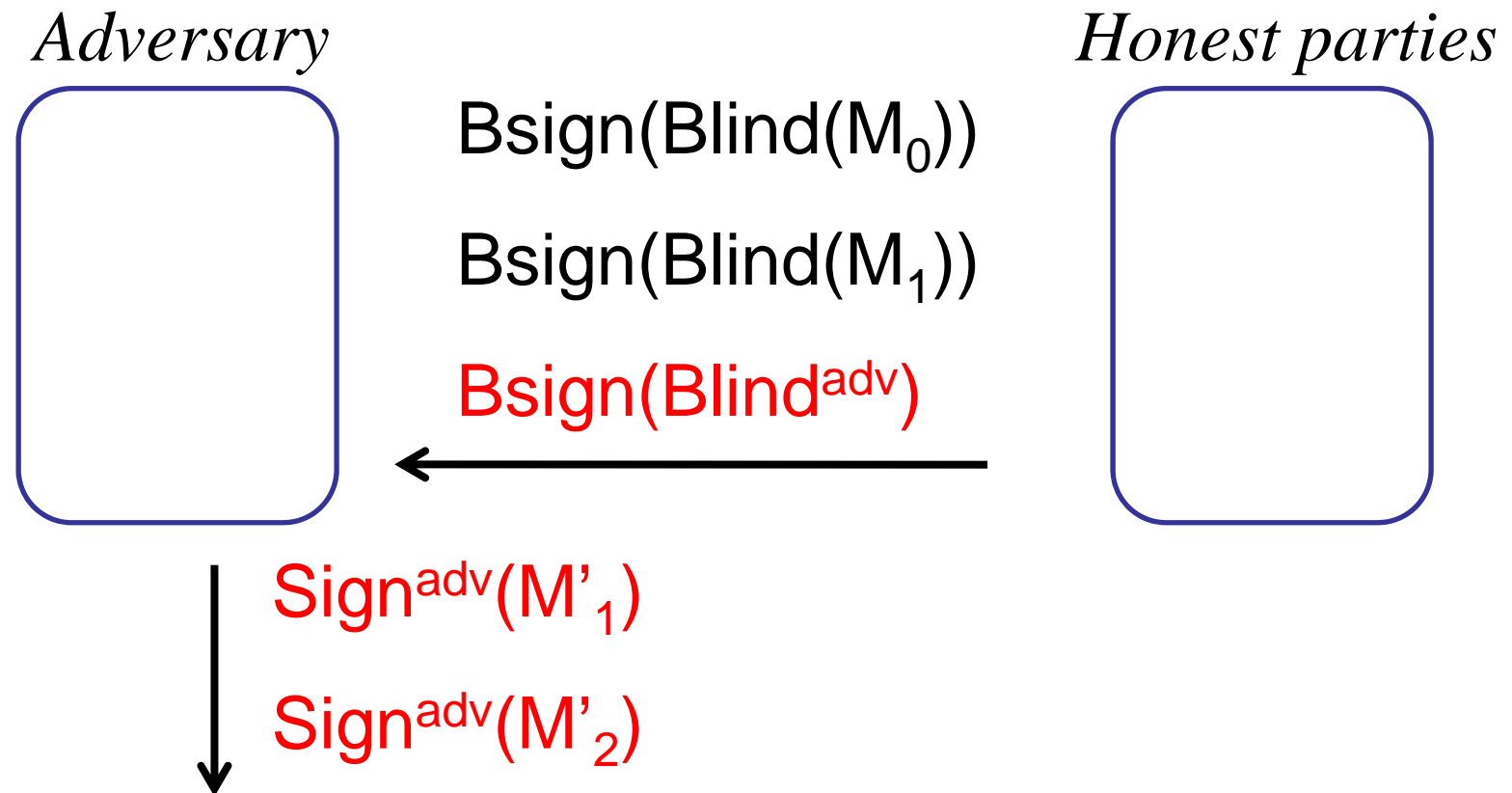
$Blind(r, m_b)$

$\dots n_b \dots$

Output  $b' = 1$  if  $n_b = n_1$

$\downarrow b'$

If the cond.2 is not satisfied





# If the cond.2 is not satisfied

*Adv. of unforgeability game*

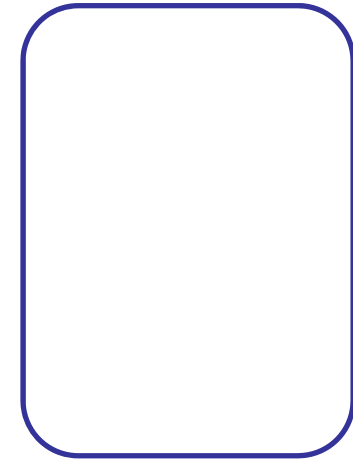
*Challenger  
(Signer)*

*Adv. prot.*

$Bsign(Blind(M_0))$

$Bsign(Blind(M_1))$

$Bsign(x)$

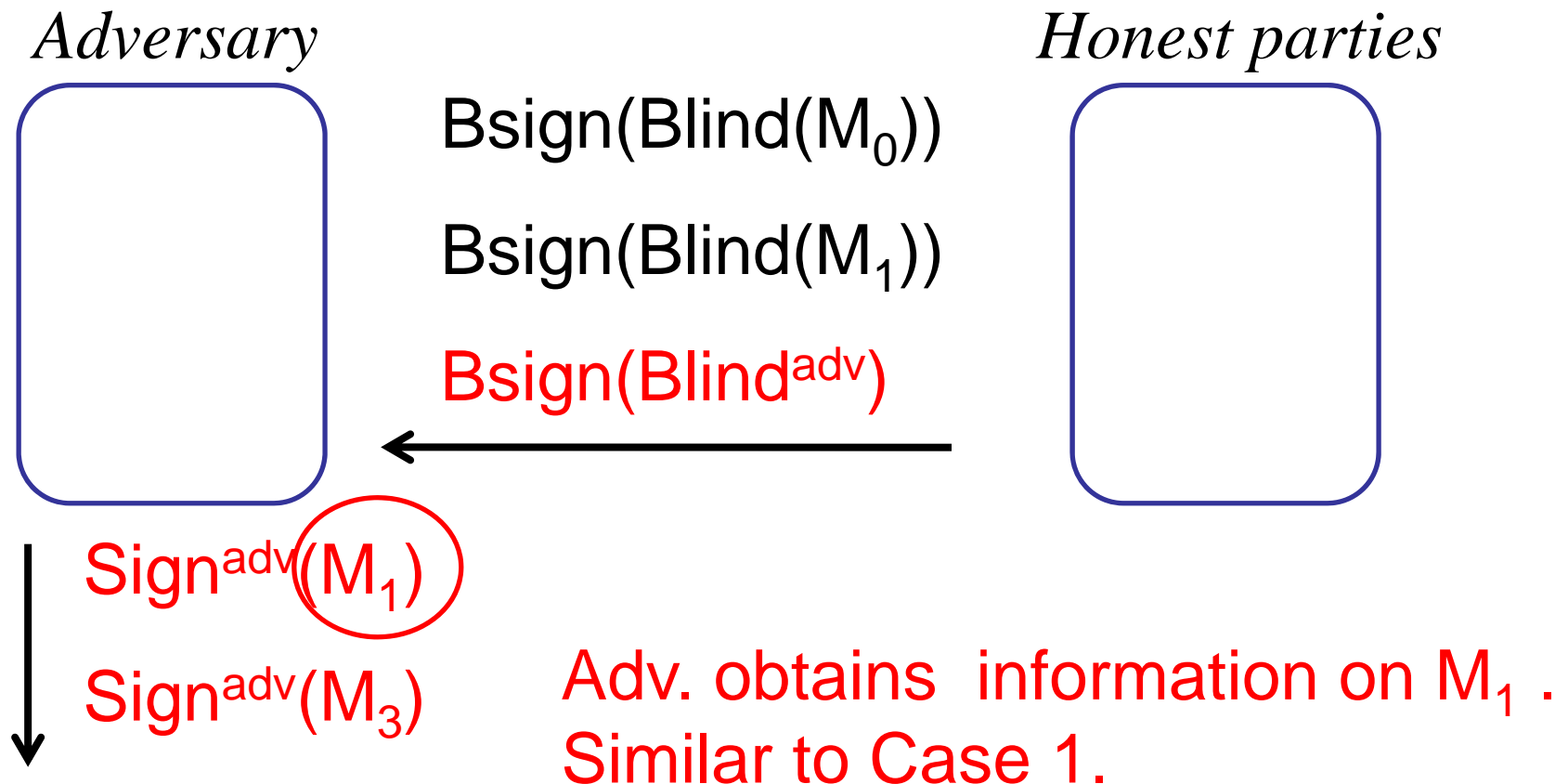


$Sign. to M_0$   
 $Sign. to M_1$

$Sign. to M'_1$   
 $Sign. to M'_2$

**Wins the unforgeability game.**

# Case 2



# Conclusion

- We have constructed a symbolic protocol model with *blind signature*
- Shown the soundness of the model with respect to the computational model where
  - Blindness
  - Unforgeabilityof blind signature are assumed.