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Equilibrium optimization of combinatorial congestion games

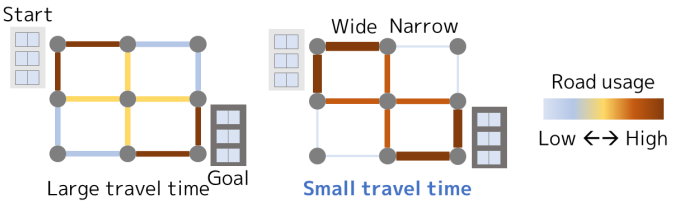
Toward uncongested infrastructures under user-equality

Abstract

In social infrastructures such as road and telecommunication networks, a link is congested and incurs more cost if many people use it. We introduce a method to compute better social design where users' cost lowers even when each user chooses a path or a combination of links selfishly. We develop a new method to compute the difference in cost when we modify the social design using a differentiable computation technique. Moreover, we compress a massive number of available paths into a data structure called a binary decision diagram, enabling us to deal with broader settings in a reasonable time. Our approach can contribute to reducing the congestion of people's flows and telecommunication networks by designing infrastructures, e.g., improving roads and expanding the bandwidth of links. Moreover, the proposed method is versatile and thus may be applicable for broader areas such as machine learning problems containing combinatorial optimization tasks.

Social Design

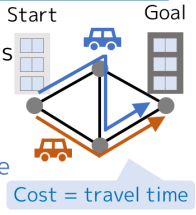
Adjustable elements in designing infrastructures (e.g., road width, speed limit, bandwidth of communication link)
We want to prevent congestion of infrastructures by adjusting them
Ex.) Adjusting road widths to decrease travel time



Congestion Game and Equilibrium State

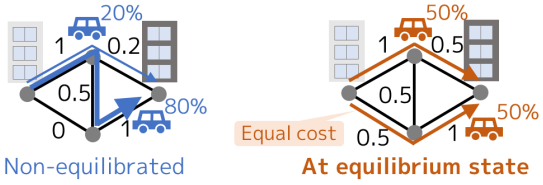
Modelling each player's selfish behavior of using infrastructures such as road networks

- There are infinitely many peoples
- Each people choose a path or a combination of links with smaller cost
- Link cost increases with higher link usage



Equilibrium state

Final result of people's trial to decrease his/her own cost
= State where every player's cost is equal and the smallest



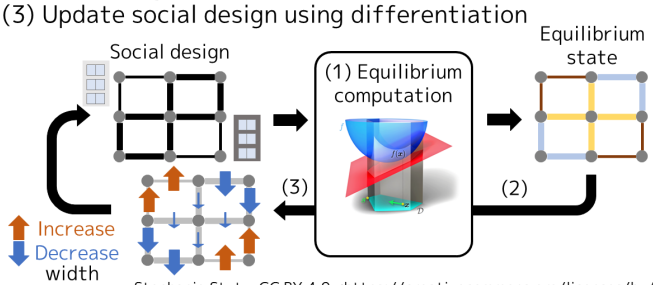
Equilibrium optimization

Computation of social design that make each people's cost smaller under some constraints such as budget

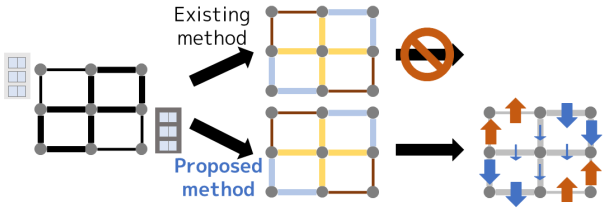
- There are so many paths that make even computing equilibrium state of a fixed social design challenging
- Moreover, we need to find out better social design

Essence of Proposed Method

Repeat:
(1) Compute equilibrium with fixed social design
(2) Compute difference of cost w.r.t. modification of social design (**differentiation**)
(3) Update social design using differentiation

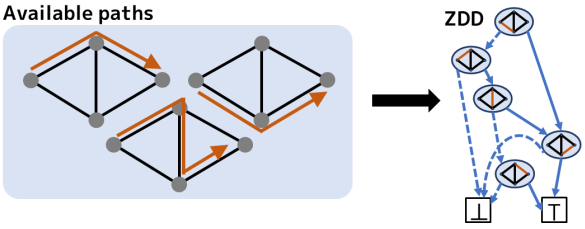


Essence 1: New equilibrium computation method that can also compute differentiation information, **enabling us to deal with broader setting**



Essence 2: Usage of zero-suppressed binary decision diagram (ZDD) that compresses available paths, **enabling fast computation**

Ex.) Representing 8 quadrillion paths with **less than 1MB**



References

[1] S. Sakaue, K. Nakamura, "Differentiable equilibrium computation with decision diagrams for Stackelberg models of combinatorial congestion games," in *Proc. 35th Conference on Neural Information Processing Systems (NeurIPS)*, 2021.

Contact

Kengo Nakamura / Linguistic Intelligence Research Group, Innovative Communication Laboratory
Email: cs-openhouse-ml@hco.ntt.co.jp