Finding new arithmetic phenomena via generalized motives

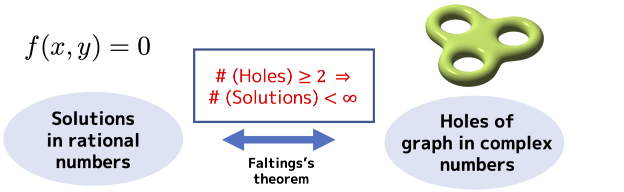
Where does the wonder of numbers come from?

Abstract

In the study of mathematics, we often find mysterious connections between two seemingly unrelated objects and phenomena. The aim of this research is to understand how these mysterious connections appear, by using the theory of generalized motives, which was developed in my previous research. We can study numbers by observing a type of shape called algebraic varieties. The theory of generalized motives enables us to continuously observe algebraic varieties from various points of view. The shapes of algebraic varieties observed from different points of view appear to be different, but they can be connected through this continuous observation. By using the theory of generalized motives, we can systematically connect seemingly different objects without relying on random luck. We anticipate that this study will accelerate the research on number theory, which underpins human activity everywhere.

Mystery in math

There are many mysterious connections in number theory. For example, the number of holes of a shape is related to the number of rational solutions of an algebraic equation.



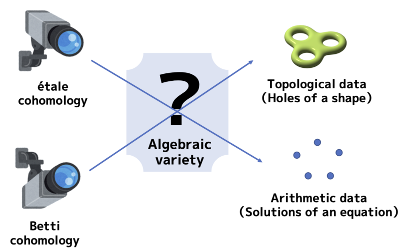
Mathematics is very good at finding a new approach to a difficult problem by connecting two things that seem to be completely different at a glance.

Wonders come from shapes

Such mysterious connections come from a type of shape called algebraic variety. We can see algebraic varieties via mathematical observation devices, i.e., cohomologies.

(Algebraic varieties are an important research subject in pure mathematics, and in applied fields such as cryptography.)

Various information, such as number of solutions and number of holes, can be obtained by observing algebraic varieties via different cohomologies.

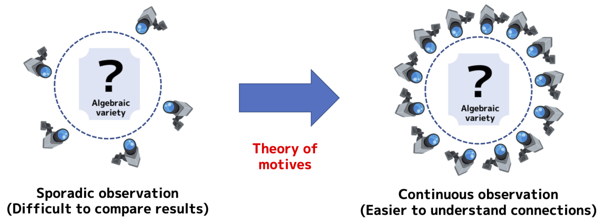


We can "see" algebraic varieties by using cohomologies.

Different cohomologies extract different types of data of algebraic varieties.

How to compare different data

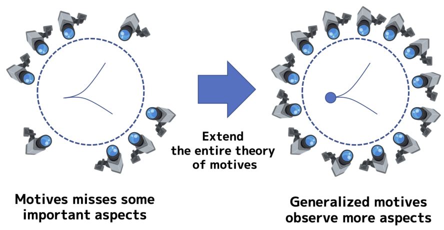
There are many cohomologies to collect different types of data, but it is not easy to find their relations just by looking at them individually. To overcome this difficulty, the theory of motives was developed to continuously observe different aspects of algebraic varieties.



Thanks to the theory of motives, mathematicians could reveal many new and deeper hidden connections.

Towards ultimate observation: generalized motive

However, some important data, such as singularity, cannot be collected by using the theory of motives. In our previous research, we have developed the theory of generalized motives to overcome this disadvantage.



Singular points reflect important properties of a shape. The theory of motives cannot observe most of the data contained in singular points.

Through high-precision observation using the theory of generalized motives, we will explore further hidden connections in the world of numbers.

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