

Abstract

Converting a program into a gate sequence is required for performing fault-tolerant quantum computation. The process of this conversion is called compilation. However, to ensure the reliability of the computation, it is usually necessary to increase the gate length, which results in a longer computational time. In this work, we have developed a compilation method that can **enhance reliability without increasing the gate length**.

We have developed an **efficient** compiler that **probabilistically generates a gate sequence** from a list of short gate sequences with an optimal probability distribution that **maximizes reliability**. Our research has shown that our compiler can **achieve the fundamental limit** of reliability, both in theory and through numerical simulations. It is imperative to develop a cutting-edge compiler to achieve the full potential of quantum computation, which has the power to revolutionize our society. We believe that our compiler provides a **practical foundation** for realizing quantum computation.

Compilation of quantum program

- We implement a **physical operation** on multiple qubits and obtain a computation result by measuring them.
- The **physical operation** consists of **unitary matrices**, which are primitive commands of a quantum program.
- Since only a very restricted subset of unitary matrices, called **elementary gates**, is implementable in a real quantum device, each unitary matrix must be compiled into a **sequence of elementary gates**.
- Compilation causes an approximation error, reducing success probability of computation. Increasing gate length can reduce the error but **increases the runtime of computation**.

Command in a program

circuit.u(θ, ϕ, λ)

correspondence

Unitary matrix

$$\begin{pmatrix} \cos \frac{\theta}{2} & -e^{i\lambda} \sin \frac{\theta}{2} \\ e^{i\phi} \sin \frac{\theta}{2} & e^{i(\phi+\lambda)} \cos \frac{\theta}{2} \end{pmatrix}$$

Example of **elementary gates**



The set of Elementary gates depends on which device we use.

Small approximation error (1%) with long runtime



Large approximation error (10%) with short runtime



The objective is to **minimize the approximation error** by using the short gate sequences.

Our approach: probabilistic compilation

Standard approach: Compile a unitary matrix into the gate sequence with the minimum error in gate sequences with a fixed length.

Our approach: **Significantly reduce the approximation error** by compiling a unitary matrix into a **probabilistically chosen gate sequence**.

Approximation error 10%



Approximation error 1%



Approximation error 11%



Approximation error 9%



I use this gate sequence every time when the same unitary matrix is used in a program.

Every time when the same unitary matrix is used in a program, it is converted into one of the following gate sequences:

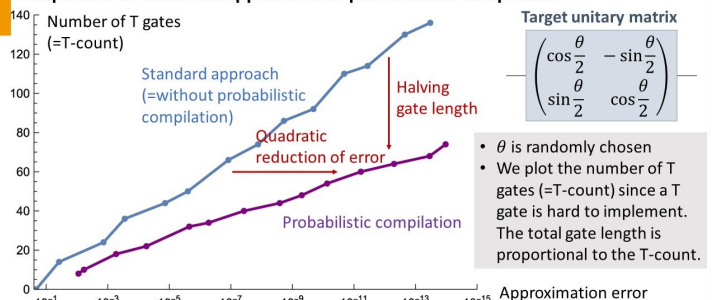
T	H	T	S	with probability 50%
H	T	H	S	with probability 30%
T	S	H	T	with probability 20%

- We have demonstrated that the error can be significantly reduced by sophisticatedly designing a **probability distribution p** for choosing a gate sequence.
- While a primitive idea of probabilistic compilation can be found in a few previous researches, we have developed the **world's first probabilistic compiler** that can efficiently compile any unitary matrix and achieve the minimum approximation error.
- We have **completely revealed the fundamental limit on the reduction rate of approximation error** owing to any kind of probabilistic compilers.

Error reduction by probabilistic compilation

- We have verified numerically that the error reduction **achieves the fundamental limit** ($\epsilon \rightarrow \epsilon^2$) when compiling a random 2x2 unitary matrix.
- This means that our probabilistic compiler can **reduce gate length by approximately 50%** to achieve a certain level of approximation error, resulting in shorter computation runtime.

Comparison of standard approach and probabilistic compilation.



References

- [1] S. Akibue, G. Kato, S. Tani, "Probabilistic state synthesis based on optimal convex approximation," *npj Quantum Information* 10, 3, 2024.
- [2] S. Akibue, G. Kato, S. Tani, "Probabilistic unitary synthesis with optimal accuracy," under review.

Contact

Seiseki Akibue
Computing Theory Research Group, Media Information Laboratory